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Rafiqul Bhuyan
California State University

Andrija Popovic
Department of Social Development

Yoshi Fukasawa
Midwestern State University

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ANALYSIS OF THE FINANCIAL INDICES OF THE NAFTA MEMBER COUNTRIES

Rafiqul Bhuyan, California State University
Andrija Popovic, Department of Social Development
Yoshi Fukasawa, Midwestern State University

This paper analyzes the efficient markets hypothesis for the major NAFTA financial indices. The results suggest that the simple return for all three indices is generally uncorrelated. The non-linear transformations of the simple return into its absolute and squared value behaved much differently however. Here, the statistics calculated provided considerable evidence to suggest that these transformations of the returns are predictable to a large degree. Ignoring the sign of the return helps greatly in predicting the direction of the series. Also, all of the series in this transformation, but one, had estimated fractional parameters that would indicate the presence of long memory. Thus, it could be concluded that volatility is a long run predictable process.

INTRODUCTION

The weak form of the efficient markets hypothesis states that given the information set at time period t, one can not predict the returns on financial assets at time period $t+1$, where the information set at time t is just the past return. More specifically, it states that the returns from one period to another are independent and one can not use past returns to predict future returns. Other forms of the theory include the "semi-strong" form where the information set includes all publicly available information and the "strong" form where the information set includes all the available information. In general, the "weak" definition of the theory is the one most commonly used for empirical testing and this is the definition that will be tested in this paper.

Literature on this subject is exhaustive. In retrospect, there is evidence both for and against the weak form of the efficiency hypothesis. In early work on financial data, Mandelbrot (1963) and Fama (1965) found that stock prices tend to be independent over time however exhibit clusters of volatility and tranquility which indicates a possible dependence within higher moments. The Box-Pierce, Ljung-Box and the variance ratio statistics developed by Box and Pierce (1970), Ljung and Box (1978) and Lo and MacKinlay (1988), respectively, are used to test if there exists dependence of the returns of a series in different moments of time.

Campbell, Lo and MacKinlay (1997) use these statistics to test for normality and predictability of the returns on the weighted CRSP (Center for Research in Security Prices) indices. They find that the autocorrelations of daily, weekly and monthly index returns are positive and significantly different from zero. In effect, this is evidence against the weak form of the efficiency markets hypothesis.

Given this result, one turns to the question of how to model the returns of financial series. In standard Box and Jenkins (1984) analysis, one often finds that financial series look like white noise and one can come to the conclusion that they follow Martingale processes. In this case, modeling the returns in a linear framework becomes uninteresting as most indices will look like random walks. For these types of models, the autocovariance function decays rapidly so that, as the time gap between observations widens, the linear relation between these observations decays rapidly. This begs the question of whether financial indices have long memory properties. Autoregressive Fractionally Integrated Moving Average (ARFIMA) models, introduced by Granger and Joyeux (1980) and Hosking (1981), exhibit long term properties. That is, observations in the distant past are correlated with observations in the far future. The autocorrelation function for these models decays slowly and thus is useful in modeling long term properties of time series data.

Studies on long-memory properties for U.S. stock prices include Greene and Fielitz (1977), Lo (1991) and Barkoulas and Baum (1996). Cheung and Lai (1995) and Crato (1994) reported results for several international indices and the G-7 countries respectively. The overall evidence suggests that stochastic long memory is generally absent from the U.S. stock markets and the international indices as well. However, there is evidence that certain individual indices do have long memory properties.

In this paper, we propose to test the weak form of the efficient markets hypothesis of the three major financial

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indices of North American Free Trade Agreement (NAFTA) members. Namely, the indices will be the Toronto Stock Exchange (TSE 300) for Canada, Standard and Poors (S&P 500) for the United States and the Mexican Stock Exchange for Mexico and the indices will be updated. The closing figure for all three series will be collected from January of 1991 to December of 2000 on a weekly basis and will be adjusted for dividends and stock splits (this data is publicly available on the internet). This analysis will be broken down into five parts.

The first part of the paper will test the normality assumption of the data using the skewness and kurtosis statistics and the Ljung-Box and Variance Ratio statistics are calculated to test for linear correlation between returns. Along with these calculations, two simple non-linear transformations of the data will also be analyzed in the same context. More specifically the absolute and squared value of the returns will be analyzed. This analysis serves the purpose of verifying whether volatility of the data is predictable. More intuitively, it tests whether large swings in the return are clustered together and predictable.

The second part of the paper will introduce the Autoregressive Fractionally Integrated (ARFIMA) Model and the asymptotic results with a general interpretation. The third part will address spectral analysis and the Geweke Porter-Hudak (GPH) semi-parametric estimator. The fourth part of this paper will deal with the estimates of the fractional parameter for the returns of the series using the GPH estimator in both absolute and squared values of returns. Finally, the fifth part of the paper will report some conclusion of the study.

Part I: Normality and Tests of Efficient Markets

In this part, it is assumed that if the market is efficient, then the price of the asset (nominal index of the series) is unpredictable, so the best predictor of the next period's price will be this period's price. Essentially, the log of the series will follow a random walk (1.1), where the errors are (NID, normally distributed) with a constant variance:

$$(1.1)\quad p_{i,t} = p_{i,t-1} + \varepsilon_{it} \qquad \varepsilon_{it} \sim i.i.d\, N[\mu, \sigma^2]$$

In above (1.1), $p_{i,t}$ is natural logarithm of index i at time t. Also, the returns on the indices of the aforementioned series will be formulated as (1.2) below.

$$(1.2)\quad r_{i,t} = p_{i,t} - p_{i,t-1} - \hat{\mu}, \text{ where } \hat{\mu} = \sum_{t=1}^{T} (p_{i,t} - p_{i,t-1})/T = (p_{i,T} - p_{i,0})/T \cdot$$

In (1.2), $r_{i,t}$ is the deviation from the mean return of index i at time t. To check if returns satisfy the normality assumption, statistics for the skewness $\hat{S}$ in (1.3) and kurtosis $\hat{K}$ in (1.4) are calculated and used to define test statistics in (1.5) and (1.6) respectively.

$$(1.3)\quad \hat{S} = [1/(T\hat{\sigma}^3)]\sum_{t=1}^{T} r_t^3, \quad \hat{\sigma} = \sqrt{1/T\sum_{t=1}^{T} r_t^2}\,.$$

$$(1.4)\quad \hat{K} = [1/(T\hat{\sigma}^4)]\sum_{t=1}^{T} r_t^4\,.$$

The test statistics are then:

$$(1.5)\quad \tilde{S} = \sqrt{T/6}\,\hat{S} \overset{a}{\sim} N(0,1)\,.$$

$$(1.6)\quad \tilde{K} = \sqrt{T/24}\,[\hat{K} - 3] \overset{a}{\sim} N(0,1)\,.$$

In testing for normality, (1.5) gives evidence on how symmetric the underlying data are and (1.6) provides evidence of kurtosis by examining how thick the tails of the distribution are. If $|\hat{S}| > 1.96$ it would be concluded with a 95% confidence level that the underlying data are not normal by virtue of skewness. It is possible the distribution of the data is symmetric but has fat tails or is not symmetric and has normal tails. For this reason, both of these calculated statistics should have a p-value less than or equal to .05 in order to suggest that the data are normally distributed at the 95% confidence level. To test against the weak form of the efficient markets hypothesis, the Ljung-Box (1.8) statistic is calculated on the basis of the autocorrelation function (1.7):

$$(1.7)\quad \hat{\rho}(k) = \frac{\sum_{t=1}^{T-k} r_t r_{t+k}}{\sum_{t=1}^{T} r_t^2}, \quad k = 1,2,\ldots,m.$$

This autocorrelation function is then used to form the Ljung-Box test statistic $\hat{Q}_m$:

$$(1.8)\quad \hat{Q}_m = T(T+2)\sum_{k=1}^{m} \hat{\rho}^2(k)/(T-k) \overset{a}{\sim} \chi^2_m .$$

The statistic $\hat{Q}_m$ tests whether there is correlation between returns in different time periods. If there is correlation between returns in different time periods, then the returns on the series are predictable. If returns are predictable, then the hypothesis of a random-walk price model is rejected and this is evidence against the efficient markets hypothesis. Here the null and alternative hypotheses are:

$$H_o : \rho(k) = 0 \ \forall \ k = 1,2,\ldots, m$$

$$H_a : \bigcup_{k=1}^{m} \rho(k) \neq 0$$

If the calculated Ljung-Box statistic is greater than the critical value of the appropriate chi-square value at the 95% confidence level, then the null hypothesis would be rejected. This would be evidence of the existence of correlation in the returns. Finally, the third statistic that is calculated is the variance-ratio statistic $\overline{VR}(q)$ given in (1.9). Like the Ljung-Box statistic, the variance-ratio statistic tests for uncorrelated returns but takes into consideration that the series may not be homoskedastic. Often it is seen that the volatility of financial data and returns changes over time. If the null hypothesis is rejected on the basis of heteroskedasticity this would be misleading. Assume that the number of observations in the sample is $T+1\{p_0, p_1, \ldots, p_T\}$. Then:

$$(1.9)\quad \overline{VR}(q) = \overline{\sigma}_b^2(q)/\overline{\sigma}_a^2,$$

in which $\overline{VR}(q)$ is the estimated variance ratio statistic, which is defined by the following:

$$\hat{\mu} = (1/T)\sum_{k=1}^{T}(p_k - p_{k-1}) = (1/T)(p_T - p_o);$$

$$\overline{\sigma}_a^2 = (1/(T-1))\sum_{k=1}^{T}(p_k - p_{k-1} - \hat{\mu})^2;$$

$$\overline{\sigma}_b^2 = (1/m)\sum_{k=q}^{T}(p_k - p_{k-q} - q\hat{\mu})^2;$$

where $m = q(T-q+1)(1-(q/T))$.

The asymptotic distribution of the variance-ratio statistic is given by (1.10). This is then normalized in the test statistic (1.11).

$$(1.10)\quad \sqrt{T}(\overline{VR}(q) - 1) \overset{a}{\sim} N(0, \sigma^2_{VR}),\ \sigma^2_{VR} = \frac{2(2q-1)(q-1)}{3q};$$

$$(1.11)\quad \hat{\psi}_q = [\sqrt{T}(\overline{VR}(q) - 1)]/\sigma_{VR} \overset{a}{\sim} N(0,1).$$

Table I gives the results of calculating $\tilde{S}, \tilde{K}, \hat{Q}$ and $\hat{\psi}$ for the returns, the absolute value of returns and the square of the returns for the three series. The p-values in each case are also reported. The skewness and kurtosis

calculations for the simple return show that the returns of all three series are not normal at the 95% confidence level. All of the series exhibit fat tails, but only the TSE index shows evidence of being non-symmetric. For the Ljung-Box statistics it can be seen that: the null hypothesis of zero correlation between the returns is accepted both at the fifth and tenth lag for the SAP index; for the TSE index, it is rejected for both of the calculated lags; and the MSE index rejects the null hypothesis at the tenth lag. In view of the results for the Box-Ljung statistic it can be concluded that the SAP index is consistent with efficient markets, but both the TSE and MSE indices seem to have returns that are predictable to some degree.

Table I: Summary, Autocorrelation and Variance Ratio Statistics

Simple Return	$\widetilde{S}$	$\widetilde{K}$	$\hat{Q}_5$	$\hat{Q}_{10}$	$\hat{\psi}_2$	$\hat{\psi}_4$	$\hat{\psi}_8$	$\hat{\psi}_{16}$
SAP Index	**0.058** [.641]	**0.912** [.000]	**9.394** [.094]	**16.508** [.086]	**-1.076** [.282]	**-0.971** [.331]	**-1.020** [.308]	**-1.026** [.304]
TSE Index	**-0.456** [.000]	**2.94** [.000]	**16.545** [.005]	**24.372** [.007]	**3.128** [.001]	**1.452** [.146]	**1.394** [.163]	**1.081** [.230]
MSE Index	**-0.068** [.580]	**1.348** [.000]	**12.130** [.033]	**15.821** [.105]	**3.123** [.001]	**3.093** [.002]	**1.589** [.112]	**1.356** [.175]
Absolute Value	$\widetilde{S}$	$\widetilde{K}$	$\hat{Q}_5$	$\hat{Q}_{10}$	$\hat{\psi}_2$	$\hat{\psi}_4$	$\hat{\psi}_8$	$\hat{\psi}_{16}$
SAP Index	**1.396** [.000]	**2.542** [.000]	**76.665** [.000]	**139.262** [.000]	**5.778** [.000]	**6.784** [.000]	**8.714** [.000]	**10.057** [.000]
TSE Index	**2.070** [.000]	**6.902** [.000]	**48.556** [.000]	**83.721** [.000]	**4.562** [.000]	**3.876** [.000]	**5.083** [.000]	**6.160** [.000]
MSE Index	**1.628** [.000]	**4.242** [.000]	**40.047** [.000]	**56.738** [.000]	**2.083** [.037]	**4.633** [.000]	**5.894** [.000]	**6.200** [.000]
Squared Return	$\widetilde{S}$	$\widetilde{K}$	$\hat{Q}_5$	$\hat{Q}_{10}$	$\hat{\psi}_2$	$\hat{\psi}_4$	$\hat{\psi}_8$	$\hat{\psi}_{16}$
SAP Index	**3.902** [.000]	**21.467** [.000]	**63.346** [.000]	**125.284** [.000]	**5.502** [.000]	**6.027** [.000]	**7.897** [.000]	**9.064** [.000]
TSE Index	**5.843** [.000]	**45.731** [.000]	**71.412** [.000]	**134.813** [.000]	**5.757** [.000]	**4.466** [.000]	**5.576** [.000]	**6.691** [.000]
MSE Index	**5.060** [.000]	**37.372** [.000]	**27.950** [.000]	**37.373** [.000]	**1.509** [.131]	**3.701** [.000]	**4.688** [.000]	**4.701** [.000]

The variance-ratio statistic yields approximately the same evidence as the Ljung-Box statistic: for the SAP index, the null hypothesis is accepted at all lags of the calculated statistic; for the TSE index, the null hypothesis is rejected at the second lag; and for the MSE index it is rejected at the second and fourth lags. Again, it would seem that the SAP index gives no evidence of predictability of returns, but both the TSE and MSE indices are predictable to some degree.

In looking at the results for the absolute value of returns and the square of returns, it is obvious that results are drastically different. Unlike the results for the simple return, which show sparse evidence of predictability, the calculated statistics for the simple non-linear transformations of the return show overwhelming evidence of predictability. The returns of all the series in the both absolute and squared values suggest rejection of the null hypothesis of normality for both the skewness and kurtosis. In regard to the Ljung-Box test, it would seem that all of the series for both transformations of the data have significant correlation for all lags. The variance-ratio statistics for all the series transformations are significant, except for the squared return of the MSE index.

The results indicate that large swings in the returns are persistent for all the series, so that volatile periods are clustered together. Even though the simple returns themselves are predictable to some degree for the TSE and MSE indices, the null hypothesis for the simple return is not rejected at an overwhelming margin thus making only crude predictions possible. This result is not surprising. If a consistent arbitrage is possible in the return of large indices, buyers would be attracted to these markets quickly and the arbitrage opportunities would vanish.

Part II: Autoregressive Fractionally Integrated Models

In standard time series analysis of the Box and Jenkins (1984) type, one is concerned in identifying an Autoregressive Integrated Moving Average (ARIMA) model. When these models are estimated, they posses autocorrelation functions that decay exponentially. Thus,

observations that are far apart become almost independent. This is known as short memory. It would be interesting to determine whether the data has long memory, but this is not possible with standard ARIMA models. Fractionally integrated models posses long-range dependence (long memory), primarily because their autocorrelation functions decay hyperbolically. In this case, observations that are far apart are still dependent on one another. This type of model is particularly interesting because of its prediction properties. (A comparison of these two autocorrelation functions is illustrated in table II3). If the return on a series is predictable for the long term, this is evidence against the efficient-markets' hypothesis, simply because the return can be forecast into the future. Returns on financial time series often follow random walks ARIMA (0,1,0) processes and are non-stationary so, first differences must be taken to achieve stationarity. This type of model can be characterized by (2.1) where x_t is the series in consideration and ε_t is the error term of the model:

$$(2.1)\quad x_t = x_{t-1} + \varepsilon_t\,, \quad \varepsilon_t \sim N[0,\sigma^2]\,.$$

To achieve stationarity, x_t must be differenced once and can be formulated as (2.2). In (2.2), B is the backward-shift operator where $Bx_t = x_{t-1}$:

$$(2.2)\quad (1-B)x_t = \varepsilon_t\,, \quad \varepsilon_t \sim N[0,\sigma^2]\,.$$

Although this model usually represents financial returns well, it is not an interesting model because accurate forecasts of the returns in the future are usually poor. Also, when the series has an autoregressive representation as in (2.3), the autocorrelation function of the series decays exponentially and long-term dependence is not possible:

$$(2.3)\quad \phi(B)x_t = \varepsilon_t\,, \ \phi(B) = (1-\phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p)\,.$$

The rapid decay of the autocorrelation function can be seen most easily in a general autoregressive (AR) model, where the AR autocorrelation function can be formulated as (2.4):

$$(2.4)\quad \rho(k) \le A\tau^k\,, \quad 0 \le \tau \le 1\,.$$

For the AR model to be stationary, it is required that $|\tau| < 1$. Also, $\tau = \max_i |r_i|$, where $\phi(r_i^{-1}) = 0$ and r_i represents all the roots of (2.3). In this case, as $k \to \infty$, the autocorrelation function τ^k approaches 0 very quickly. This is the short-memory property of regular autoregressive moving-average (ARMA) models. By this result, observations in the distant past are independent of observations in the future so making very long-term predictions is often inaccurate and unreliable. To introduce an Autoregressive Fractionally Integrated Moving Average (ARFIMA) model we ma write (2.2) in the more general format:

$$(2.5)\quad (1-B)^d x_t = \varepsilon_t\,, \ \varepsilon_t \sim N[0,\sigma^2]\,.$$

In this model, we may think of the d parameter as taking any real value. If d is integer valued, this is just a special case of a regular ARMA model. If the d parameter has a value between 0 and 1, then the model is an ARFIMA model. To know what happens to the formulation of (2.5), a Taylor series of $(1-B)^d$ expansion is taken around $B = 0$.

$$(2.6)\quad (1-B)^d = 1 - dB + \frac{d(d-1)}{2!}B^2 - \frac{d(d-1)(d-2)}{3!}B^3 + \cdots\,.$$

From (2.6) it can be seen that the differencing operator $(1-B)^d$ is an infinite polynomial expression in d and B. Granger and Joyeux (1980) and Hosking (1981) demonstrate that, by the binomial theorem for non-integer powers, (2.6) may be also expressed as (2.7):

$$(2.7)\quad (1-B)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} B^k\,, \ \binom{d}{k} = \frac{d(d-1)\cdots(d-k+1)}{k!}\,.$$

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If this expression is applied to the x_t variable, expression (2.5) becomes:

$$(2.8)\quad (1-B)^d x_t = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} B^k x_t = \sum_{k=0}^{\infty} A_k x_{t-k} = \varepsilon_t \,.$$

Further, the autoregressive component of (2.8) can be expressed in terms of the gamma function (2.9). Also, x_t can be expressed as an infinite moving average as in (2.10) and expressing the infinite-order moving average polynomial in terms of the gamma function (2.11):

$$(2.9)\quad A_k = (-1^k)\binom{d}{k} = \frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} \,.$$

$$(2.10)\quad x_t = (1-B)^{-d} \varepsilon_t = \sum_{k=0}^{\infty} B_k \varepsilon_{t-k} \,.$$

$$(2.11)\quad B_k = \frac{\Gamma(k+d)}{\Gamma(d)\Gamma(k+1)} \,.$$

Granger (1980), Granger and Joyeux (1980), and Hosking (1981) show that the characteristics of these models are very useful in modeling time series. This is because the autocorrelation function of the ARFIMA model decays hyperbolically and x_t is stationary for values for $d \in (-\frac{1}{2}, \frac{1}{2})$, (see Hosking, 1981). In particular, the autocorrelation function can be approximately formulated as (2.12):

$$(2.12)\quad \rho_f(k) \sim \frac{(-d)!}{(1-d)!} k^{2d-1} \,.$$

To compare the long-memory autocorrelation function to that of an AR(1) model, Table II is reproduced from Campbell, Lo and MacKinlay showing with the decay of an AR(1) model with $\phi = .5$, $d = -\frac{1}{3}$ and $d = \frac{1}{3}$. Here, both the fractionally differenced series $d = \frac{1}{3}$ and the AR(1) models have an autocorrelation of 0.5 for the first lag, but after this, they behave very differently. While the AR(1) model has a correlation of 0.000 at the 25th lag, the fractionally differenced model, $d = \frac{1}{3}$ has a value of 0.173 at the 25th lag. (Note when $d < 0$ this may be called the antipersistence case and $d > 0$, the long-range dependence case.) From this it can be seen that, when a series exhibits long-range dependence, observations are correlated with one another even at large lags. This being the case, it becomes clear why such a process is useful in modeling returns. When modeling returns with regular ARMA models, the autocorrelation function decays very quickly and thus predictions are only useful for the short run. When considering the ARFIMA model the autocorrelation function decays close to zero only after a long time, thus making predictions becomes possible in the long run.

Table II: Autocorrelation function for a fractionally differenced process and an AR(1)

Lag K	$\rho_f(k)$ $d = \frac{1}{3}$	$\rho_f(k)$ $d = -\frac{1}{3}$	$\rho(k)$ $\phi = \frac{1}{2}$
1	0.500	-0.250	0.500
2	0.400	-0.071	0.250
3	0.350	-0.036	0.125
4	0.318	-0.022	0.063
5	0.295	-0.015	0.031
10	0.235	-0.005	0.001
25	0.173	-0.001	$2.98\text{x}10^{-8}$
50	0.137	$-3.24\text{x}10^{-4}$	$8.88\text{x}10^{-16}$
100	0.109	$-1.02\text{x}10^{-4}$	$7.89\text{x}10^{-31}$

A series that is stationary exhibits a variance that is positive and finite. Considering a simple AR(1) model such as:

$$(2.13)\quad x_t = \phi x_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim N[0, \sigma^2] \,.$$

If we take the unconditional variance of (2.13) and solve for the variance of x_t, we obtain (2.14):

$$(2.14)\quad (Var(x_t) = \phi^2 Var(x_{t-1}) + Var(\varepsilon_t),\quad Var(x_t) = \frac{\sigma^2}{1-\phi^2}.$$

If expression (2.14) is looked at, it becomes clear that the variance of x_t is both positive and finite if and only if $\phi \in (-1,+1)$. By the same token, we can calculate the variance of x_t, when x_t is a fractional process. First, to calculate the variance of x_t conveniently, the series can be first expressed as an infinite moving average process as in (2.10). Granger (1980) expresses the moving average (MA) weights as (2.15) for large k and an appropriate constant A:

$$(2.15)\quad B_k \approx Ak^{d-1} \text{ for } k \geq 1.$$

Now an MA(∞) model (2.16) can be expressed with B_k, $k \geq 1$, given exactly by (2.15):

$$(2.16)\quad x_t = \sum_{k=0}^{\infty} Ak^{d-1}\varepsilon_{t-k} = A\sum_{k=1}^{\infty} k^{d-1}\varepsilon_{t-k} + \varepsilon_t.$$

If the variance of (2.16) is taken on the assumption that the error term has a mean of 0, a constant variance (2.17) results:

$$(2.17)\quad Var(x_t) = Var(A\sum_{k=1}^{\infty} k^{d-1}\varepsilon_{t-k} + \varepsilon_t) = A^2\sigma^2(\sum_{k=1}^{\infty} k^{2(d-1)}\varepsilon_{t-k} + 1).$$

In this case, for the variance of x_t to be finite and positive $\sum_{k=1}^{\infty} k^{2(d-1)}$ must converge to a finite number. From the theory of infinite series, it is know that $\sum_{k=1}^{\infty} \frac{1}{k^s}$ converges to a finite number for $s > 1$, and thus for x_t to have a finite variance $-2(d-1) > 1$, so the result is that $d < \frac{1}{2}$. Thus it follows if $d > \frac{1}{2}$, the variance of x_t diverges and becomes infinite so implying that the series would be non-stationary. Hosking (1981) also demonstrates that when $-\frac{1}{2} < d < 0$, x_t is invertible with infinite order moving average representation (2.8) and when $0 < d < \frac{1}{2}$, x_t is a stationary process and has infinite moving average representation (2.10).

Part III: Geweke Porter-Hudak Special Regression

Since the fractional model (2.6) is a polynomial in d and B it is not obvious how d should be estimated. A proposed method is the classical rescale range (R/S) method, but this does not have a well-defined distribution. Moreover the distribution is sensitive to changes in the underlying data generating process. The R/S method finds the existence of long-memory too often. Geweke and Porter-Hudak (1983), GPH83, proposed the spectral regression method and Lo (1991) proposed the modified R/S statistic, these are both semi-parametric methods of estimating d. Both of these estimators are consistent and have well-defined distributions. In this paper the Geweke Porter-Hudak (GPH) estimator will be used because it is a consistent estimator which has a sound distribution and is computationally non-intensive. It is necessary to switch attention from the time series domain to the frequency domain. The GPH estimator uses properties of the periodogram and spectral density to estimate the d parameter. An estimate of the d parameter is found by regressing the periodogram on a constant and an explanatory variable that is a function of the sin of angular frequencies, for specific frequencies (see (3.7)-(3.9). An introduction to spectral analysis will first be described to introduce frequency domain analysis. "Spectral Analysis" is equivalent to time domain analysis based on the autocovariance function, but provides an alternative way of looking at a series which is helpful in identifying certain characteristics of the data. This method is especially useful in looking at filters for the data such as the fractional model. Suppose x_t is a series that is stationary with a mean of 0, then the spectral density of x_t can be expressed as:

$$(3.1)\quad f(\lambda)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\cos(-k\lambda)\gamma(k)\cdot$$

Similarly, the autocovariance function may also be expressed as a particular function of the spectral density:

$$(3.2)\quad \gamma(k)=\int_{-\pi}^{\pi}\cos(k\lambda)f(\lambda)d\lambda\cdot$$

The spectral density also has certain properties that are similar to the autocovariance function such as $f(\lambda)=f(-\lambda)$. Also, for $f(\lambda)\geq 0$ on the interval $(-\pi,\pi]$ the function is unique. If f and g are two spectral densities corresponding to the autocovariance function $\gamma(\cdot)$, then:

$$\gamma(k)=\int_{-\pi}^{\pi}\cos(k\lambda)f(\lambda)d\lambda=\int_{-\pi}^{\pi}\cos(k\lambda)g(\lambda)d\lambda\,.$$

To understand how the spectral density models a particular series, an example of white noise is taken. Assume that $x_t\sim N(0,\sigma^2)$, then $\gamma(0)=\sigma^2$ and $\gamma(k)=0$ for all $|k|>0$. Then the spectral density is:

$$(3.3)\quad f(\lambda)=\frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\gamma(0)=\frac{\sigma^2}{2\pi},\ -\pi\leq\lambda\leq\pi\,.$$

It can thus be seen that the spectral density of a white noise process is a constant. This means that each frequency in the spectrum contributes equally to the variance of the process. When the process that generates x_t is a stationary AR(1) model, $x_t=\phi x_{t-1}+\varepsilon_t$ with $0<\phi<1$, then the spectral density is given by (3.4). This is dominated by primarily low frequencies, because the autocorrelation function (ACF) is positive and large at the first lag:

$$(3.4)\quad f(\lambda)=\frac{\sigma^2}{2\pi}\cdot\frac{1}{(1+\phi^2-2\phi\cos(\lambda))}\cdot$$

When $-1<\phi<0$, then the AFC is large and negative at the first lag and the spectral density is dominated by high frequencies. In this manner, the spectral density has specific characteristics for each process that is generated by the x_t variable. The model $x_t=(1-B)^{-d}u_t$ can be expressed in the frequency domain by (3.5), where u_t is a stationary linear process, which is bounded away from zero, finite and continuous on the interval $[-\pi,\pi]$.

$$(3.5)\quad f(\lambda)=|\psi(e^{i\lambda})|^2 u_t=\psi(e^{i\lambda})\cdot\psi(e^{-i\lambda})u_t,\ \psi(e^{i\lambda})=(1-e^{i\lambda})^{-d}\,.$$

Equation (3.5) may be expressed as:

$$(3.6)\quad f(\lambda)=\frac{\sigma^2}{2\pi}\{2[1-\cos(\lambda)]\}^{-d}\,.$$

GPH83 rearrange (3.6) and express the spectral density function of x_t as:

$$(3.7)\quad f(\lambda)=(\frac{\sigma^2}{2\pi})\{4\sin^2(\frac{\lambda}{2})\}^{-d}\text{ ; in which}$$

$$2[1-\cos(\lambda)]=2[\sin^2(\lambda/2)]\,.$$

Taking the natural logarithm of (3.7) yields:

$$(3.8)\quad \ln[f(\lambda)]=\ln(\frac{\sigma^2}{2\pi})-d\ln\{4\sin^2(\frac{\lambda}{2})\}\cdot$$

Suppose that series x_t is $\{x_1,x_2,\ldots,x_T\}$. Let the harmonic ordinates be $\lambda_{j,T}=2\pi j/T\quad(j=0,\ldots T-1)$ and $I(\lambda_{j,T})$ denote the periodogram at these ordinates. From this, Geweke and Porter-Hudak rearrange (3.8), then evaluate it at $\lambda_{j,T}$ yielding:

$$(3.9)\quad \ln\{I(\lambda_{j,T})\} = \ln\{\frac{\sigma^2}{2\pi} f_u(0)\} - d\ln\{4\sin^2(\lambda_{j,T})/2\} + \ln\{\frac{f_u(\lambda_{j,T})}{f_u(0)}\} + \ln\{\frac{I(\lambda_{j,T})}{f(\lambda_{j,T})}\};$$

where $f_u(\lambda)$ is the spectral density of u_t. Equation (3.9) appears like a least squares equation with $ln\{I(\lambda_{j,T})\}$ being the dependent variable, $ln\{(\sigma^2/2\pi)f_u(0)\}$ the intercept, $ln\{4\sin^2(\lambda_{j,T})/2\}$ the explanatory variable and $ln\{I(\lambda_{j,T})/f(\lambda_{j,T})\}$ the disturbance term. Further, the term $ln\{f_u(\lambda_{j,T})/f_u(0)\}$ becomes very small and can be ignored when the harmonic frequencies are near to 0. In this case, GPH83 propose an estimator of $-d$ as the slope coefficient of the regression of $ln\{I(\lambda_{j,T})\}$ on $ln\{4\sin^2(\lambda_{j,T})/2\}$ and a constant for a sample of size $G(T)$ where $G(T)$ is a function of T [see Geweke and Porter-Hudak, 1983]. In considering the validity of (3.9), it must be noted that the distribution of $ln\{I_u(\lambda_{j,T})/f_u(\lambda_{j,T})\}$ is i.i.d of the Gumbel type which has a mean of –C and a variance of $\pi^2/6$. The value of C is Euler's constant, .57721. This argument is based on asymptotic theory which leads to certain restrictions on $G(T)$ if the sample is to be used to estimate the slope coefficient [see Geweke and porter-Hudak, 1983]. The regression equation (3.2) can be expressed as:

$$(3.10)\quad ln\{I(\lambda_{j,T}\} = B_0 + B_1 ln\{4\sin^2(\lambda_{j,T})\} + u_{j,T},\ j = 1,...G(T).$$

In this regression, B_1 is the ordinary least squares estimator and $ln\{I(\lambda_{j,T})\}$ is the periodogram at the frequencies $\lambda_{j,T=2\pi j/T}$ in a sample of size T. Assuming that the properties of $G(T)$ are satisfied, then, $(B_1 + d)/\sqrt{\{v\hat{a}r(B_1)\}} \xrightarrow{a} N(0,1)$, where $v\hat{a}r(B_1)$ is the usual estimated variance of B_1. Also, the known theoretical variance of the error term in the spectral regression (3.10) is $\pi^2/6$. In addition to proving asymptotic normality, GPH83 prove consistency for $d < 0$. [Later, Robinson (1990) proved consistency for $d \in (0, 0.5)$]. To test their method, Geweke and Porter-Hudak choose ordinate values that were consistent with the theory of $G(T)$. To this end, they found that $T^v, v = .5$ is a relatively good choice for estimating the slope. If v is too large, then the contribution of $ln\{f_u(\lambda_{j,T})/f_u(0)\}$ can no longer be neglected. In regards to the experiments that were conducted, GPH83 found that using the theoretical value of the error term, $\pi^2/6$, is considerably more reliable than the estimated variance. Also, they suggest that 100 observations are sufficient for reliable estimation.

Table III: Estimated Long Memory Parameter

Simple Return	$\hat{d}$	S.D.
SAP Index	-0.098	0.112
TSE Index	-0.317	0.112
MSE Index	-0.441	0.112
Absolute Value	$\hat{d}$	S.D.
SAP Index	0.478	0.112
TSE Index	0.679	0.112
MSE Index	0.205	0.112
Squared Return	$\hat{d}$	S.D.
SAP Index	0.488	0.112
TSE Index	0.463	0.112
MSE Index	0.157	0.112

Part IV: Estimation Results

In estimating the long-run parameter for the data, the $G(T)$ function is set to $T^{.5}$ and the reported standard error of the coefficients is the true asymptotic value given in (4.1) below. The estimated d parameter along with the standard deviation for the simple, absolute and squared returns are reported in table III.

$$(4.1)\quad \sqrt{Var(\hat{d})} = \frac{\pi}{\sqrt{6}} \sqrt{\sum_{i=1}^{\sqrt{T}} 4\sin^2\left(\frac{\lambda_i}{2}\right)}, \text{ where } \lambda_i = 2\pi i / T$$

The term d is estimated to be significantly different from 0 in all three cases. The estimated d for the simple return of all of the series is negative which may indicate that some anti-persistence exists in the data. The return on the SAP index has an estimate of d that is close to zero. This would suggest that the returns on the Standard and Poors Index very closely resemble a white noise process and the market is efficient. The returns on the TSE and MSE indices, however, show a much larger negative persistence. This could possibly be some indication that the simple returns on these indices may be predictable, evidence that would be against the efficient markets hypothesis. However, this is very difficult to reconcile with efficient markets theory. If returns are predictable on such a large scale, arbitrageurs would notice such an opportunity and take advantage of the market and soon the arbitrage opportunity would disappear. Even though the standard deviations of the estimates indicate that negative persistence of the TSE and MSE indices is significant, the results could be due to small sample bias.

If the behavior of the absolute value and squared simple returns are looked at, they have quite different results. For the absolute value of the simple return, all of the three estimates of d are positive. Both the SAP and MSE index have a parameter value that would suggest that the absolute value of the return possesses a long memory property. The TSE index, however, has an estimated value of d that is above .5 and this is evidence that the series, when transformed in this way, is not stationary. Also, the TSE index has a value of d that is close to .5 so a case for non-stationarity cannot be ruled out as a possibility. The MSE index, is however, the only index that has a value of d that is both significantly above 0 and below .5 so it would seem to possess long memory property.

When the simple return is squared, all of the indices have an estimated d that is positive and below .5. The SAP and TSE indices have parameter values that are again close to .5 so a case for non-stationarity cannot be ruled out. Again, in this case, the MSE index is the only one that has an estimated d parameter that is both significantly above 0 and below .5, so the square of the return possess long memory properties.

By these results, it would seem that the simple return on the SAP index has no long memory properties; however the TSE and MSE indices have results that suggest they could possibly possess negative persistence. When the sign of the return is ignored, then most of the series seemed to possess long memory properties.

CONCLUSION

This paper analyzed the efficient markets hypothesis for the major NAFTA financial indices. The results of the first part of the paper shows that the simple return for all three indices is generally uncorrelated, although some of the test statistics support the rejection of this hypothesis by a very slim margin. In estimating the fractional parameter for the three series, there was no evidence of any long memory patterns for the S&P index. However the estimates for the TSE and MSE indices indicate that the simple return on these two indices might possess anti-persistence. This is probably due to small sample bias. In general, the three series seem to support the "weak" form of the "Efficient Markets" theorem and there is most likely no arbitrage opportunity in the indices.

The non-linear transformations of the simple return into its absolute and squared value behaved much differently however. Here, the statistics calculated provided considerable evidence to suggest that these transformations of the returns are predictable to a large degree. Ignoring the sign of the return helps greatly in predicting the direction of the series. Also, all of the series in this transformation, but one, had estimated fractional parameters that would indicate the presence of long memory. Thus it could be concluded that volatility is a long run predictable process.

Future studies on these series should perhaps concentrate on forecasting competitions between standard time series and fractional models. This type of research could indicate whether the negative persistence found in the TSE and MSE indices is useful for prediction. Another area of future research may include the effect on market efficacy of the implementation of the NAFTA itself. One would expect an increase in market efficiency, but no empirical results are currently available.

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Rafiqul Bhuayn is an assistant professor of finance at California State University Sacramento. He received his Ph.D. in finance from Concordia University. He has published articles in national and international journals.

Andrija Popovic is an economist at social policy research group, Department of Social Development, Quebec, Canada. He received his MA in economics from Concordia University. He has published articles on Canadian government's social development related issues.

Yoshi Fukasawa is professor of economics at Midwestern State University. He received his Ph.D. in economics from Kansas State University. His research focus is on the Texas economy and international economics. He has published in Journal of Economics, Southwestern Journal of Economics, and Texas Business Review.