A Compilation of Published Research In Elementary Arithmetic
Since 1931

Alfred C. Birdsell
Fort Hays Kansas State College

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A COMPILATION OF PUBLISHED RESEARCH IN
ELEMENTARY ARITHMETIC SINCE 1931

being

A thesis presented to the Graduate Faculty
of the Fort Hays Kansas State College in
partial fulfillment of the requirements for
the Degree of Master of Science

by

Alfred C. Birdsell, B. S.
Fort Hays Kansas State College

Date July 21, 1950

Approved

Major Professor

Chairman Graduate Council
DEDICATION

This thesis is dedicated to my patient and faithful wife, Lorraine, and to my two children, Joyce Louise and Warren Charles, to my parents and my wife's mother, all of whom have made innumerable sacrifices to make my education possible.
ACKNOWLEDGMENTS

The writer of this thesis appreciates the patient, thoughtful guidance of Dr. Robert T. McGrath, Professor of Education, Fort Hays Kansas State College, and the timely suggestions of Dr. F. B. Streeter, Librarian and Secretary of the Graduate Council.
PREFACE

The writer of this thesis has always been interested in improvement of instruction in arithmetic. It is believed that the present investigation will be quite beneficial in showing what progress has been made in teaching methods and will serve to direct the teacher of elementary arithmetic toward a renewed interest in the best possible instructional procedures. This work consists of a survey of articles of published research in the field of elementary arithmetic. They have been classified under many headings to facilitate their use.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. RESEARCH PERTAINING TO ARITHMETICAL FUNDAMENTALS</td>
<td>5</td>
</tr>
<tr>
<td>Concepts of Pre-School Children</td>
<td>5</td>
</tr>
<tr>
<td>Addition and Subtraction</td>
<td>6</td>
</tr>
<tr>
<td>Fractions</td>
<td>8</td>
</tr>
<tr>
<td>Multiplication</td>
<td>9</td>
</tr>
<tr>
<td>Division</td>
<td>10</td>
</tr>
<tr>
<td>Compilation, Drill and Review</td>
<td>13</td>
</tr>
<tr>
<td>Errors and Relative Difficulty</td>
<td>16</td>
</tr>
<tr>
<td>III. PROBLEM SOLVING AND THE PSYCHOLOGY OF LEARNING</td>
<td>18</td>
</tr>
<tr>
<td>Psychology and Analysis of Problem Solving</td>
<td>18</td>
</tr>
<tr>
<td>Development of Concepts</td>
<td>20</td>
</tr>
<tr>
<td>Improvement and Generalization</td>
<td>21</td>
</tr>
<tr>
<td>IV. CORRELATION WITH OTHER ABILITIES</td>
<td>24</td>
</tr>
<tr>
<td>General Intelligence</td>
<td>24</td>
</tr>
<tr>
<td>Vocabulary of Arithmetic</td>
<td>25</td>
</tr>
<tr>
<td>Arithmetic in Reading Material</td>
<td>26</td>
</tr>
<tr>
<td>V. METHODS OF INSTRUCTION</td>
<td>28</td>
</tr>
<tr>
<td>Analysis of Teaching Methods</td>
<td>28</td>
</tr>
<tr>
<td>Self Instruction</td>
<td>29</td>
</tr>
<tr>
<td>Social Utility Method of Instruction</td>
<td>30</td>
</tr>
<tr>
<td>Instruction Through Co-Curricular Activities</td>
<td>32</td>
</tr>
<tr>
<td>Weakness in Teaching Methods</td>
<td>33</td>
</tr>
<tr>
<td>CHAPTER</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>VI. CURRICULUM AND COURSES OF STUDY</td>
<td>35</td>
</tr>
<tr>
<td>Trends on Emphasis in Textbooks</td>
<td>35</td>
</tr>
<tr>
<td>Grouping and Grade Placement of Materials</td>
<td>36</td>
</tr>
<tr>
<td>Evaluation of Textbooks</td>
<td>38</td>
</tr>
<tr>
<td>VII. TESTING AND TESTING TECHNIQUE</td>
<td>41</td>
</tr>
<tr>
<td>Readiness Tests</td>
<td>41</td>
</tr>
<tr>
<td>Methods of Scoring</td>
<td>42</td>
</tr>
<tr>
<td>Measuring Understandings</td>
<td>43</td>
</tr>
<tr>
<td>Reliability of Tests</td>
<td>44</td>
</tr>
<tr>
<td>Individual Differences and Achievement</td>
<td>44</td>
</tr>
<tr>
<td>Diagnostic Testing</td>
<td>45</td>
</tr>
<tr>
<td>Remedial Measures</td>
<td>46</td>
</tr>
<tr>
<td>VIII. GENERAL RESEARCH</td>
<td>48</td>
</tr>
<tr>
<td>IX. CONCLUDING SUMMARY AND RECOMMENDATIONS</td>
<td>51</td>
</tr>
<tr>
<td>BIBLIOGRAPHY</td>
<td>53</td>
</tr>
</tbody>
</table>
CHAPTER I

INTRODUCTION

Many articles are published each year pertaining to elementary arithmetic. Some of these may very properly be designated "mere opinion" while others may very properly be classified as "research". Because of the large number of such articles appearing in print pertaining to some aspect of arithmetic, no doubt a service may be rendered to teachers of the subject by evaluating and classifying them as research or otherwise. From this situation grew the problem of this thesis, the title of which may be stated as follows:

A Compilation of Published Research in Elementary Arithmetic Since 1931. Such a guide should be useful to teachers of arithmetic in classrooms wherever they may be.

Research has been defined in various ways by different investigators. Whitney defines research as a state of mind—a friendly, welcoming attitude toward change.1 C. C. Crawford at the University of Southern California explains research after this fashion: Research is simply a refined technique of thinking, employing specialized tools, instruments and procedures in order to obtain a more adequate solution of a problem than would be possible under ordinary means. It starts with a problem, collects facts, analyzes these critically and reaches decisions based on the actual evidence. It involves

original work rather than exercise of mere opinion. It evolves from
genuine desire to know rather than a desire to prove something. 2
According to Webster, it is careful or critical inquiry or examina-
tion seeking facts or principles, diligent investigation in order
to ascertain something. 3 Likewise, in the Dictionary of Education
by Good, research is said to be of such a nature that others would
get the same result. 4 But for the purpose of this study the term
research will be used to mean a study which starts with a problem,
collects facts, analyzes these critically and reaches decisions
based on the obtained facts. It will usually consist of an exper-
iment in which the sample is general enough that others would get
the same results.

This assembly began with the year 1932 because December, 1931,
marks the close of a similar survey by Guy T. Buswell and C. H.
Judd, who published a monograph 5 in 1925 based on research in elemen-
tary arithmetic up to that time. Buswell issued seven annual
supplements to the original monograph, bringing the total number of
articles and books reviewed to 697. The final supplement included
research published and reported during the entire year of 1931.

2. Ibid., p. 20.

3. A. Merriam Webster, Webster's Collegiate Dictionary,

4. Carter V. Good, Dictionary of Education (New York:

5. G. T. Buswell, Survey of Research in Elementary Arithmetic
Since that time, Leo J. Brueckner, as well as Buswell, continues to report through the *Elementary School Journal* on selected references in elementary arithmetic, but does not distinguish between research articles and those of critical discussion which do not bear the earmarks of research. This fact is clearly set forth by a number of authorities in the field. The number of non-research articles is not only increasing, but it is believed that many articles which have been considered research should not be classified as such. Buswell challenges the field to recapture the spirit of true research that we might progress in effective teaching of arithmetic.

The problem is limited to scientific investigations reported in the field of elementary arithmetic since 1931. Algebra and geometry, even though sometimes taught in the elementary school, will not be included. While completeness is attempted, caution will be taken to include only those articles which in the opinion of the author can be classified as research.

The survey method was employed in securing the data. Since it was not possible to obtain and appraise every investigation in the field of elementary arithmetic over such a long period, some articles were excluded on the ground that the titles suggested they...

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were not research. The selected references published in the Elementary School Journal were helpful in discovering many of the citations in this study. Also, the references given in the Education Index include articles of research together with numerous articles which did not measure up to the desired standard. Other sources and bibliographies also containing reports were used to complete this survey.
CHAPTER II

RESEARCH PERTAINING TO ARITHMETICAL FUNDAMENTALS

More research has been conducted pertaining to the fundamental operations than any other phase of arithmetic. Not only does the study of the four fundamental operations—addition, subtraction, multiplication and division—make up a large part of the elementary arithmetic curricula, but it is the basis for all other phases of arithmetic. It is for this reason that it is placed first in this compilation.

Concepts of Pre-School Children

Children learn quite a few quantitative ideas before they begin formal study of arithmetic. As these ideas are very important in determining later progress, it is highly necessary that the primary teacher be familiar with the amount and nature of the child's understanding of the number system. The following studies reveal many astounding facts relative to concepts of pre-school children.


Reports a study of the comparative effectiveness of counting and grouping in the development of certain mathematical concepts.

Grant, Albert, "An Analysis of Number Knowledge of First Grade Pupils According to Levels of Intelligence." See Correlation with Intelligence.

Koenker, R. H., "Arithmetic Readiness at the Kindergarten Level," *Journal of Educational Research*, XLII (November,
Addition and Subtraction

Addition and subtraction are very closely related and are often taught together. It has also been convenient to study the problems of addition and subtraction together in a number of cases. The work in addition and subtraction of fractions presents problems of a special nature and is taken up in a separate section.


A review of a larger study in third grade arithmetic.

Brownell, W. A., "An Experiment on Borrowing in Third-Grade Arithmetic," Journal of Educational Research,
Experiment reveals that there are merits to various methods, but consideration of the value of the instruction is at stake.


Checking should not be taught with the process, but after the pupil understands its value.


Familiarity with the combinations upon entering school is a factor in learning.


Reports an investigation with college students with various conditions of drill in addition.


Extensive correlations and difficulty ratings expressed, but recommends further study.

Wilburn, D. Banks, "A Method of Self-instruction for Learning the Thirty-Six Addition Combinations with Sums from Eleven to Eighteen and Their Corresponding Subtraction Facts in Grade II," Mathematics Teacher, XXXVIII (October, 1945), 246-51.

A study of the achievement of sixty-four pupils with a method of self-instruction for learning the addition combinations.

Shows evidence of many methods of teaching subtraction, but recommends that we stick to one method to eliminate confusion of students.

Fractions

Under the heading of fractions is listed research the type of research studies pertaining to decimals, ratios and percentages as well as those pertaining to common fractions. It is believed that this arrangement will facilitate the use of this compilation of research in analyzing the best methods of classroom instruction.


Gives data on types of errors in fractions in a sixth-grade class, and suggests four methods of improving teaching of fractions.


Difficulties in percentage encountered by adults may be an indication of poor teaching.


Analyzes difficulties and errors in decimals.


Large itemized list of difficulties shown by experiment and tells where more emphasis needs to be placed.

Shows errors and difficulties that the school should investigate.


Evidence is presented showing that present methods of learning fractions lacks retention.


Studies four concepts of fractions, presenting teaching methods on the subject.


We need to abandon continued drill on some decimals, but at the same time appreciate their existence.


A study of frequency of occurrence of fractions.

Multiplication

Only those research studies dealing with the teaching of the multiplication facts are considered in this section. In view of the fact that multiplication is a definite part of the long division process, it is not necessary to study multiplication as such alone.


Considers sixty-eight types of errors in multiplication made by 2,577 pupils in grades four, five and six.


The difficulties encountered in the learning process are different from those in the final process.

Division

Division is much more complex than any of the other three basic operations. It not only involves the use of multiplication and subtraction, which are memory processes, but in long division it is necessary to estimate the quotient in advance. Since setting the decimal point is also a part of the division process, the division of decimals is given consideration in this section.

Brueckner, L. J., and Harvey O. Melbye, "Relative Difficulty of Types of Examples in Division with Two-Figure Divisors," Journal of Educational Research, XXXIII (February, 1940), 401-11.

A good article on relative difficulty of examples in grades 5B and 6A.

Dahle, Casper O., "The Verbal Thought and Overt Behavior in Children During Their Learning of Long Division." See Problem Analysis.


A detailed classification and analysis of estimates of quotients in long division.

Grossnickle, F. E., "Comparison of Achievement of Pupils Who Are Good and Poor in Learning with a Two-Figure Divisor," Journal of Educational Research, XXXIV (January, 1941), 303-51.

A study of ninety-four children shows no great difference in the two groups either in native ability or in eventual achievement.

Grossnickle, F. E., "Constancy of Error in Learning Division with a Two-Figure Divisor," Journal of Educational Research, XXXIII (November, 1939), 189-196.

A study based on 231 pupils of grade four indicating the fact that error is not constant, but invites further investigation.


Listings of cues in division problems are presented from textbooks for grades three to six.

Grossnickle, F. E., "Division Facts and Their Use in Estimation of Quotient with Two-Figure Divisor," Elementary School Journal, XLV (June, 1945), 569-74.

Study shows the importance of the division facts for use in estimating the quotient.

Grossnickle, F. E., "Estimating the Quotient by Two Methods in Division with a Three-Figure Divisor," Elementary School Journal, XXXIX (January, 1939), 352-56.

A comparison of the relative accuracy of the apparent and the increase-by-one method.

Grossnickle, F. E., "An Experiment with a One-Figure Divisor in Short and Long Division," Elementary School Journal, XXXIV (March and April, 1934), 496-506, 590-99.

An experiment which suggests that only the long division
form should be taught.


There was no appreciative difference in the results of the two methods used in this experiment.

Grossnickle, F. E., "How a Method of Scoring a Test in Division Affects the Score," Elementary School Journal, XL (January, 1940), 366-70.

Shows that a score for correct estimate is also necessary for instructional purposes.


Identifies twenty-one different kinds of errors, some of which are constant.


Investigation shows that some methods are much preferred over others.


Extensive study of examples in workbooks shows need for improvement.


Survey study of examples in textbooks.

Grossnickle, F. E., "Some Factors Affecting a Test Score in Division of Decimals." See Test Scoring.

Grossnickle, F. E., "Types of Errors in Division of Decimals," Elementary School Journal, XLII (November,
A study of two hundred pupils in grades six to nine, showing common errors and how to avoid them.


Analysis of thirteen sources of difficulty in the long division process with valuable suggestions for the benefit of the learner.

Koenker, Robert H., "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division." See Correlation with Intelligence.


Suggested methods for estimating. Recommends rounding numbers for greater meaning.


Experiment with pupils in grades four to twelve showed greater accuracy by three-fourths of the students when long division was the standard method.


A detailed study survey of examples in long division.


Examination of transformations from one unit to another.

Computation, Drill and Review

The question often arises as to the optimum amount of drill and review. A great deal of research is being conducted relative to
these important aspects of instruction. No doubt this was brought on in part by the need of the Armed Forces for speedy learning or review of many fundamental operations.


A follow up study which indicates that remedial work is of little value in many cases.


A comparison of scores on the Cleveland Survey Arithmetic Test given in St. Louis in 1916, and again in the year 1938.

Braverman, Benjamin, "Does a Year's Exposure to Algebra Improve a Pupil's Ability in Arithmetic?" The Mathematics Teacher, XXXII (November, 1939), 301-12.

A study suggesting that algebra helps improve the arithmetical ability of high school pupils.


Shows that drill does not guarantee child growth for several given reasons.


Discusses deficiencies in arithmetic according to tests in Navy. Proposes refresher course in high school.


Reports the results of successive usage of an arithmetic test during grades seven and eight.

A summary of investigations relating to drill as well as a discussion of the use of drill.

Grossnickle, F. E., "Practice Material in Estimation of the Quotient in Long Division Found in Current Textbooks." See Division.


Some courses contribute more than others. Recommends remedial work as part of algebra.


Reports the results of an experiment with various instructional procedures.


A discussion of errors as a basis for research and an evaluation of a few research papers.

Wilson, G. M., and Gertrude L. Hanley, "For Basic Drill in Arithmetic - What Norm or Average is Satisfactory?" See Test Scoring.


Gives results of test comparing the effects of blackboard and lantern-slide presentation of arithmetic combinations.
Errors and Relative Difficulty

Extensive studies have been made concerning the many types of errors in the fundamental processes and the implication with regard to teaching arithmetic in the elementary school. Along with this is the relative difficulty of the various processes and how their order in a given situation affects the difficulty.


The position of a process has a decided influence on the difficulty of the entire problem.


Subtraction-addition is more difficult than addition-subtraction.


Describes with reliability a technique to determine relative difficulty of problems.


Errors are more constant in division than in other operations. Important for diagnosis.


Shows how the two types of errors (chance and constant) are related to various operations as well as to maturity.

Analyzed the errors made by a group of teachers on a survey test in computational arithmetic.


A study to show where more stress needs to be placed.


Borrowing and changing to common denominators proved to be major difficulties.


A brief article based on arithmetical weaknesses found in the Army Air Forces. Accuracy was the first glaring weakness.

See Problem Concept Development.
CHAPTER III

PROBLEM SOLVING AND THE PSYCHOLOGY OF LEARNING

The analysis and solution of written problems is without question one of the more difficult parts of the arithmetic program. This chapter deals with the very nature of problem solving and then a later chapter will be given to methods of instruction which will have a bearing both on the fundamental processes and the solution of written problems.

Psychology and Analysis of Problem Solving

Real problem solving is not a trial and error procedure; rather, it is a process based on analysis. For this reason it is necessary for both the teacher and the student to fully understand the basic operations involved in the solution of the problem. There are many factors which contribute to the success of the student in problem solving.

Brownell, W. A., "Rate, Accuracy and Process in Learning," Journal of Educational Psychology, XXXV (September, 1944), 321.

A good article on educational psychology based on tests and interviews pertaining to learning multiplication combinations.


Drawing on data of previous research, this article shows the contribution of the crutch in borrowing in subtraction.
Buswell, G. T., "Types of Learning and General Conditions Affecting Learning," Review of Educational Research, IX (June, 1939), 274-84.

A review of research on the learning process.


An elaborate study of the learning process with an implication in arithmetic.

Dexter, C. E., "Analysis of Written Problems in a Recent Arithmetic Series." See Evaluation of Textbooks.


Identification and description of the essential processes of problem solving.


An important summary of the various methods of problem solving.


A study of the various factors contributing to problem-solving ability.


Study of improvement of 117 pupils in grades seven and eight when problems were rearranged.


Experiment in fifth grade to determine the value of specific training in reading verbal problems.

Identifies nine aspects of problem-solving ability.

Sutherland, Ethel, "One-Step Problem Patterns and Their Relation to Problem Solving in Elementary Arithmetic," *Teachers College Record*, XLIX (April, 1948), 492-93.

An analysis of 15,000 verbal problems in terms of the thought-patterns involved.


A study to determine which of the fundamental processes were needed most in solving functional problems.

Development of Concepts

The development of concepts is an essential part of the problem solving technique. It consists of the original knowledge of quantitative data together with formation of mental pictures of the essential elements in the problem at hand.


There is no significant difference between the effectiveness of the conventional type and the imaginative type problem.


An extensive study showing that time problems should be based on the out-of-school experience of the children.


Shows great need for oral work at various levels of difficulty.


Common fractions should be limited to halves, fourths, and eighths and increase emphasis on decimals.


Experimental study of the relative perceptual value of concrete number situations in primary grades.


Central ideas in teaching problem-solving must be developed.

White, Helen M., "Does Experience in the Situation Involved Affect the Solving of a Problem?" Education, LIV (April, 1934), 451-55.

A study based on one thousand sixth-grade children. Shows value of experience.

Improvement and Generalization

The pupil's ability to generalize plays a very important part in the successful solution of problems. Along with this the student needs to learn to use various aids and devices to help him in problem-solving. Practice in these will no doubt bring improvement in the solution of written problems.

Mastery of arithmetic calls for learning by both repetition and insight. Shows need for understanding aids to learning.


Results on Stone Reasoning Test show that best problem-solving material comes from children's own environment.


An experiment to discover a means of improving the problem-solving ability of upper-grade pupils.


Experiment shows practice in estimating improves results.


A condensed article reporting reactions to fifty-four generalizations in arithmetic.


Grades five and six reports analyzed for rapid, average and slow pupils.


Lists many factors and discusses the age range in which they are the most pronounced.

A review of the nature of generalization and the need for more study in this field.


Shows that guidance in problem-solving is a requisite.


Children working together were more successful in problem solving than were control children working alone.
CHAPTER IV

CORRELATION WITH OTHER ABILITIES

Ability in arithmetic is closely associated with other knowledge and abilities. For instance, a child who comes to school with experience in number concepts is able to do much better work than the child who has had limited experience in numbers. Likewise, general intelligence is a primary factor in determining ability to do good quality work in arithmetic.

General Intelligence

For efficient instructional and scoring purposes, it is desirable to correlate the material with the general intelligence of the students. There is also a close relationship between this concept and readiness testing. Students learn best with material of optimum difficulty, so it is necessary for efficient instruction to understand the general ability of the students.


The responses of a large number of first-grade pupils in a test of number knowledge.

Grossnickle, F. E., "Comparison of Achievement of Pupils Who Are Good and Poor in Learning with a Two-Figure Divisor." See Division.

Koenker, Robert H., "Certain Characteristic Differences Between Excellent and Poor Achievers in Two-Figure Division," Journal of Educational Research, XXXV (April, 1942), 578-86.
Analysis of responses of 180 pupils on situations in division.


Considers the interrelations of ability in silent reading, power in the fundamentals, problem solving, and general intelligence.

Vocabulary of Arithmetic

This unit deals with the vocabulary of elementary arithmetic used by pupils and teachers. It shows that there is a positive correlation between vocabulary or experience and ability to understand and solve arithmetic problems.

Brueckner, Leo J., and Harvey C. Melbye, "Relative Difficulty of Types of Examples in Division with Two-Figure Divisors," Journal of Educational Research, XXXIII (February, 1940), 101-11.

An extensive attempt to determine relative difficulty with recognition of previous studies.


Classified list of seventeen arithmetical functions in the daily lives of children.


Report of difficulty of arithmetical terms.


A comparison of arithmetical vocabularies of students and teachers.

Value of vocabulary toward the improvement of problem-solving ability set forth.


A study of vocabulary difficulties in certain textbooks.


A study of the social usefulness of the arithmetic vocabulary in four investigations.


Experiment shows important evolution of concepts at various age levels, pre-school and primary.


Arithmetic terms used in seventy-seven classrooms in California with frequencies of each work.


Gives data showing knowledge of various units of measure.

Arithmetic in Reading Material

Another important aspect of instruction is the correlation with other formal subjects such as reading. In this way opportunity
is provided for learning arithmetic in a more natural situation. Several studies have been conducted to determine how much opportunity for learning arithmetic concepts are provided in the reading material of elementary school children.


A study of arithmetical vocabulary in reading textbooks. Shows need for more correlation.


Meaning in arithmetic suggests actual significance of work and also the immediate social setting. Reading is very important in solving arithmetic problems.


A very good research article on the need for reading skills.


An experiment showing why children can learn to read numbers very quickly in the first grade.


Reviews related studies and presents the results of analyses of books and materials assigned in Grades III-VIII.
CHAPTER V

METHODS OF INSTRUCTION

There are a number of outstanding methods of teaching certain aspects of arithmetic, each of which has its own merit. Several methods have been studied recently in an effort to improve instruction. A review of the various methods of the past with their weaknesses together with an understanding of the prevailing methods of today should be of considerable worth to any teacher of mathematics.

Analysis of Teaching Methods

The many methods of instruction are compared in this section together with some methods that cannot be classified in the few major divisions following. This also includes some of the history of methods of instruction.


A study of the development of meaning in arithmetic and its value to the child.


An interpretation of the significance of Gestalt psychology for the teaching of arithmetic.
Dickey, John W., "Readiness for Arithmetic," Elementary School Journal, XL (April, 1940), 592-98.

A presentation of the research on readiness in the light of the methods of instruction.


An analysis of sixty-four experiments reported between 1911 and 1940 pertaining to methods of teaching arithmetic.


Opinions of teachers about fifty percent in agreement with research and judgment of experts. A valuable study for determining need for study in workshops and similar undertakings.


The abacus is useful in exploring number meanings and was an indispensable part of the equipment of arithmeticians for over a thousand years.


Shows decline of drill and the rise of arithmetic as a social study.

Self Instruction

A number of aids and devices have been used to make it possible for the student to learn certain habits in arithmetic without the constant aid of an instructor. Among these are special rules for
learning the fundamental combinations in addition and subtraction. Workbooks offer another opportunity for the student to work under his own power.


An analysis of seventy-three workbooks in arithmetic followed by an extensive bibliography.


A timely article on the use of "crutches". Points out both values and dangers in the use of such devices.


A limited experiment with seventy-two pupils in grade one. Shows value of a simple method of self-instruction.

Social Utility Method of Instruction

It is commonly believed that students are more interested in problems taken from their own environment than they are in those which seem to have no meaning to them. However, it is not easily determined just what constitutes meaningfulness to children. At the same time there must not only be meaning but motivation. For this reason a few experiments have been conducted to determine how much social arithmetic is desirable.


A presentation of the meaning theory in arithmetic, followed by an answer to some objections to teaching for meaning.


Presents a list of problems from the field of social science to be used in courses of study in arithmetic.


Problems that children select are most useful.

Gorman, Frank H., "An Experiment in Integrating Seventh and Eighth Grade Science and Mathematics," Science Education, XXVII (December, 1943), 130-34.

A comparative study of effectiveness of teaching integrated mathematics and science. Little difference noted.


Examination of business concepts used in thirteen textbook series. These concepts were not adequately provided for.


A lower grade experiment in isolating number experiences and giving them preference over social uses.


Very good to increase the child's insight into the problem.

Indicates that the arithmetic now taught in grade three exceeds the child's needs for addition and that subtraction covers most of the needs at this age.


Attempt to improve teaching by making it interesting and putting it in life situations.


A story of the evolution of arithmetic in the out-of-school life of children beginning with the work of Binet in 1890.


A good article showing the absurdity of many written problems.


Social utility theory defended as a method of teaching arithmetic.

Instruction Through Co-curricular Activities

Just as there is evidence to support the social utility method of instruction, there is justification for teaching some aspects of arithmetic through the activity program. It is believed that a good teacher must reap the good from all these methods as they apply to the situation at hand.

A continuation of a very favorable experiment on learning decimals in socially real situations.


Presents the results of a study showing that fundamentals in fractions are efficiently learned when they arise in activity units in a random order without further practice.


A classroom study showing that children can learn the fundamental operations in an informal atmosphere such as the activity program.


Study of 342 children in the third grade who learned multiplication by playing a game.

Weakness in Teaching Methods

The above studies have all been of a positive nature and classified accordingly, but in this section entitled "weaknesses" an attempt has been made to include those studies which emphasize the low qualities of some methods. In this way it is possible to present a better view of the facts pertaining to methods of instruction.

Grossnickle, F. E., "Illustrations in Arithmetic Textbooks," Elementary School Journal, XLVII (October,
1946), 84-92.

There is need for more functional pictures with more problems accompanying each illustration.


A controlled experiment of 1212 pupils in the upper grades showing abilities in which they are weak. Individualized group instruction is more effective than conventional group instruction.


Shows that early deficiency in number continues unless pupils are definitely taught the facts and the processes of number.


The method of teaching has a very important part in the learning process.


Points out that the schools need to have clearer objectives for the students.


Shows that usual method is time wasted.
CHAPTER VI
CURRICULUM AND COURSES OF STUDY

Under the above title are gathered together those reports of research which deal with the evolution of the textbook showing the trends of emphasis and the best arrangement of materials. In earlier chapters materials and methods have been stressed, but in this chapter the emphasis is on content and arrangement to facilitate instruction in the classroom.

Trends on Emphasis in Textbooks

Through the years and in different schools of thought, emphasis has been placed on different parts of the arithmetic curricula. As these trends are carefully studied, we should arrive at a reasonable balance of stress on various topics. These studies cover a long period of time, in some cases going back as far as the textbooks of 1790, so the errors of the past should be quite evident. With this continued follow up, the trend should be more easily seen.


Lists items receiving more or less attention in fifty-five textbooks between 1860 and 1933.


An interesting piece of classroom research showing the
need for a practical arithmetic course for those students not going beyond high school.


(Fifth Period—1911-1940), Vol. 19, No. 6, 1943, 45 pp.


The best available factual data for a history of the development of arithmetic textbooks in America during the past 150 years. The present publication is culmination of a series of previous investigations.


A good discussion of the trend toward "meaning" for the child.


Review of three studies, one her own, on the extent of usage of metric system in countries where it is legally compulsory.

Grouping and Grade Placement of Materials

Another important aspect of teaching arithmetic consists of grouping the material in such a way that it is easily understood. Some processes being much more complex than others,
curriculum designers and textbook writers should arrange the material according to the maturity of the pupils. As the responsibility falls back on the teacher to arrange the material in the best possible order, teachers will find this contribution vital to improved instruction.


Describes three sources of research supported by a lengthy bibliography.


A study of the kinds of research on child development as applied to curriculum building.


A review and a very extensive discussion of some of the results of research on grade placement in arithmetic.


Shows optimum placement of topics.


Determined mental age at which seventy-five percent of children mastered certain processes.

Ruch, G. M., "Relative Difficulty of the One Hundred Multiplication Facts with Special Reference to Textbook Construction," See Multiplication.

Topics of several textbooks classified.


Investigation shows failure is due to introducing material before children are ready for it.


An attempt to determine the mental-age level at which pupils may understand certain topics in arithmetic.

Wetherington, Julia, "Grade Placement of the Content of Arithmetic," Elementary School Journal, XXXVII (September, 1936), 41-46.

An analysis of ten courses of study.

Evaluation of Textbooks

Textbooks are often scrutinized in an effort to determine the books containing the best qualities. On the other hand, it may be desirable to analyze a given series of textbooks to determine to just what extent various functions are taken care of without reference to comparison with other books. The studies which follow are therefore of varying nature, but all bear upon an analysis or evaluation of textbooks.


Brueckner, Leo J., and James A. Irving, "A Technique for Comparing the Difficulty of Problems in Textbooks

Gives a measuring device for textbook committees on relative difficulty of problems.


Experiment showing that many problems in textbooks are pointless.


Analysis of ten series of texts published between 1925 and 1930. Gives relative amount of space given to each type of problem.


Grossnickle, F. E., "Cues in Division Problems Given in Nine Representative Textbooks in Arithmetic." See Division.

Grossnickle, F. E., "Illustrations in Arithmetic Textbooks." See Weakness in Teaching Methods.


Analysis of twelve textbooks for space given to measurement.


Statements supported by research for setting up program.


Shows that many problems have little permanent value.
O'Rourke, Everett V., and Cyrus D. Mead, "Vocabulary Difficulties of Five Textbooks in Third-Grade Arithmetic." See Vocabulary of Arithmetic.


Analyzes five textbooks in relation to four prevailing theories of instruction.
CHAPTER VII
TESTING AND TESTING TECHNIQUES

Testing is being given much more consideration in the schools today than formerly. We have come a long way with the testing program, but much remains to be accomplished in this field. A wise teacher will want not only to experiment for finding better methods, but will want also to keep up with the results of research in this important program. It is believed that the citations following will be of great help to teachers in improving testing techniques.

Readiness Tests

Testing for readiness is one of the newer concepts in the testing program. It consists of determining the relative difficulty of various items and then correlating this with the mental and educational age of the pupils. Preliminary work in this field is already done by the textbook writers and courses of study, but with increased individual differences and other variable factors it is always necessary for teachers to consider readiness testing. Suggestions are presented here for devising and using readiness tests.


Illustrates procedures employed in devising and validating a readiness test.

Brueckner, L. J., "Development and Validation of an Arithmetic Readiness Test," Journal of Educational Research,
Detailed analysis of a good readiness test.

Dickey, John W., "Readiness for Arithmetic." See Analysis of Teaching Methods.


A study of the concepts which children recall. Useful in determining readiness.


Examination of several types of readiness tests together with the relationship between predicting success in arithmetic as compared with other subjects.


Readiness tests proven to be valuable in instruction.


Describes test for first grade for determining readiness for items.


First half deals with reading and last half with arithmetic readiness. Review of studies and bibliography.

Methods of Scoring

Scoring is certainly an important part of the total testing program. It has been shown that the method of scoring makes a
decided difference in the results to be obtained. Unless the score is as nearly accurate as possible and we can interpret it, our testing program is worthless. For this reason experiments have been conducted to help make the score more meaningful.

Grossnickle, F. E., "How a Method of Scoring a Test in Division Affects the Score." See Division.


Test items should be arranged and relatively easy.

Wilson, G. M., and Gertrude L. Hanley, "For Basic Drill in Arithmetic, What Norm or Average is Satisfactory?" The Mathematics Teacher, XXXII (April, 1939), 175-178.

Drill work must be perfect in order to be of value to the student. Norms are unreliable for this work.

Measuring Understandings

This phase of measurement is quite new and is as yet not fully developed. It would certainly be wonderful if teachers of mathematics could be able to determine to what degree pupils understood and appreciated certain ideas in arithmetic. Often times accuracy and other factors come between understanding and the test score.


Analysis of a test for measuring the child's ability to reason. Correlation is positive between reasoning and other mathematical skills.

Improving the program in arithmetic by showing that old formal tests do not show the child's appreciation for the subject.

Reliability of Tests

Too often teachers give tests and assume that the results are infallible. While there are a large number of standardized tests on the market, it is often more practicable to give a good teacher-made test. Teachers then should be familiar with methods of determining validity and reliability of their tests.


An analysis of several good arithmetic tests.


Individual Differences and Achievement

Arithmetic is a subject in which individual differences is most pronounced. If children fail to see a given process, they are from that time on retarded. This makes it challenging to a teacher of arithmetic to see that the needs and abilities of all the pupils are cared for.


I. "Ability to Differentiate Extraneous Materials from Needed Arithmetic Facts." Retarded students of a given mental age are inferior in problem solving to normal children.

II. "Understanding Arithmetic Processes." Shows that mentally retarded children of a given mental age are
not consistent in their solutions.


Extensive tabulation of results of Stanford Intermediate Arithmetic Test when given to about a quarter of the sixth-grade enrollment of Indiana.


A good article of ability of varying kinds.


An experiment showing that ability to discriminate and match numbers increases with age.

Olander, Herbert, and others, "Predicting Arithmetic Achievement." See Readiness Tests.


Distribution of scores on the Wilson Addition Process Test.

Diagnostic Testing

Among the many kinds of testing is diagnostic testing, which consists of giving a test that is so especially designed that one is capable of discovering specific weaknesses not determined by ordinary means.

Shows that pupils do not make the same errors repeatedly on the same test; therefore, one test is usually unreliable.


Tests should consist of four or more items of each type arranged in rows.


Shows that at least three examples of each type is necessary for reliable diagnosis.


Discusses effect on the score by method employed in problem solving.


Review of previous research on this important question, with recommendations.


A general report of individual diagnosis tests given in the upper grades.

Remedial Measures

After difficulties are located, it is necessary to take remedial measures to correct the deficiency. Sometimes that work comes in the form of review and drill in the fundamentals while on
other occasions it is special training which seems to lead to more indirect results. There is a difference of opinion as to the value of this work. No doubt this is an area in which more work needs to be done in order to show appreciable results.


Reports that remedial work is of less value than usually supposed.

Elair, Glenn Myers, "Remedial Arithmetic in Senior High School," The Mathematics Teacher, XXXVI (December, 1943), 346-50.

Discusses the subject on the basis of a previous research study in 166 high schools.


Guiler, W. S., and H. B. Hoffman, "Improving Computational Habits of Ninth Grade Pupils," Educational Administration and Supervision, XXIX (September, 1943), 345-56.

Experiment on remedial needs of a large number of students.


Remedial work should be interesting and very beneficial.


Results of experiment in remedial arithmetic in the junior high school.
CHAPTER VIII

GENERAL RESEARCH

This chapter contains reports of research which are either general in scope so that they could not be rightfully classified under any special topic in the field of arithmetic as well as those dealing with research itself. Many of the titles listed below are those of the nature of this compilation except that they cover only a very limited period of time or a special aspect of teaching arithmetic. A few of these studies proved useful to the author for the purpose of securing information about other research articles which were unavailable for review for this compilation. Several bibliographies contain a large number of titles which have been classified as quite important even though they do not fall into the category of true research. Should the reader be interested in some of those titles, they may be easily obtained from the following reports.


A history of research with list of problems and needs yet to be studied.


Classified review of eighty-two studies of the previous three years.

A review of 142 articles and books of research and otherwise.


A review of research over a three year period.


Fifty titles in arithmetic considered over a three year period.


This annual report, usually by Buswell, carries a good bibliography of research and critical discussion related to instruction. While this is a very select list, the emphasis is on material to improve instruction and is not all true research.


Shows that much inferior material is coming out and challenges the field to recapture the spirit of true research.


A review of a few valuable studies in mathematics, grades seven to twelve.


An analysis of previous articles by Dickey and Wheat regarding zero.


A bibliography and review of twelve studies.


A number of ideas are covered, ranging from order of difficulty of fundamentals to curriculum and social utility methods.

Wilson, Guy M., "Bibliography Written Problems in Arithmetic," Education, LIV (April, 1934), 480-83.

Unannotated bibliography of eighty-five references.


The literature published from October 1, 1931 to July 1, 1934 is reviewed.
CHAPTER IX

CONCLUDING SUMMARY AND RECOMMENDATIONS

The number of articles appearing in print each year pertaining to arithmetic is increasing rapidly, but in spite of this fact, the number of these articles which bear the earmarks of research is by no means increasing. It has been discovered in reviewing numerous articles for this compilation that almost without exception the majority of the titles listed in the bibliographies bear dates prior to 1930. This fact indicates that fewer contributions of significant worth are appearing as well as the fact that the few studies which otherwise might be of real value are unnoticed amid the large volume of material of lesser importance.

It is believed that this present compilation should serve the purpose of keeping before the reader those experiments and findings which are new and of value to all concerned. While a detailed analysis of each piece of research is left to be accomplished by the coming generation, it is firmly believed that this present preliminary classification is of utmost importance to teachers and others interested in securing the best information regarding certain aspects of arithmetic. Since a large majority of the experiments covered in this study were conducted in the classroom situation, the results lend themselves most readily to classroom teaching.

It is the contention of the writer that compilations of this nature in other fields of endeavor should likewise be made. It is also believed that future study in the field of elementary arith-
metic would be justified. While completeness is striven for, it is many times impossible for the examiner to obtain and review all the articles which appear in less well known periodicals. For this reason the writer welcomes further research along these lines. At the same time, future study could keep the index current.

A survey of research in arithmetic from early time to the end of the year, 1925, with complete bibliography.


A dictionary of concepts pertaining to educational research and education in general.


Another survey of research in arithmetic following the one by Buswell and similar to it except that the emphasis is on teaching the subject.


A technical guide for research activity.

Periodicals


Views the problems of research and challenges the field to recapture the spirit of true research.


