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A Study Of Retention Of Percentage

Fred Dellett
Fort Hays Kansas State College

A STUDY OF RETENTION OF PERCENTAGE

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Fred Dellett, A. B.
Fort Hays Kansas State College

Date July 25, 1946. Approved Robert J. McGrath
Major Professor

F. D. Albertson
Chairman Graduate Council

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CHAPTER I

INTRODUCTION

Civilization today is becoming so complex with the changes in our various social institutions that we have come to consider the mathematical ability of our youth as never before. The years covering World War II made increasing demands upon science and mathematics. During this period - a period of progress - accuracy carried great significance. Methods of teaching have changed. Thorndyke very clearly points this out when he says, "The newer methods emphasize the processes which life will require and the problems which life will offer."[1]

During the author's experience as a mathematics teacher, various problems of teaching the unit of percentage in both grade and high school presented themselves. One of the areas in mathematics, which has shown frequent misunderstanding and lack of fundamental knowledge, is that of percentage. Reeve, in an address before the National Council of Teachers of Mathematics made this significant statement: "About percentage, for instance, one finds astonishing ignorances in most unexpected quarters."[2]

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1. Edward L. Thorndyke, The New Methods in Arithmetic. (New York, Rand McNally and Co., 1911), p. 1.

2. W. D. Reeve, "Attacks on Mathematics and How to Meet Them." National Council of Teachers of Mathematics, 11th Yearbook, p. 3.

Statement of Problem

The problem of this thesis was to determine by means of a test the retention of the fundamentals of percentage as shown by beginning eighth grade pupils, who were taught percentage the previous year in the seventh grade.

The fundamentals of percentage are here considered as the changing of percents to decimal fractions or common fractions, the reverse procedures, and the three types of problems arising from the relation, base times rate equals part. These quantities, base, rate, and part, are represented in the example, 300 X 12% = 36, respectively.

Because of the inherent relationship of decimal fractions and percent, many of their common uses are overlapping. Percentage has been taught as something distinct and separate from the decimal fraction. "The introduction of percentage was due to the need of a common denominator in comparing numbers, especially, in the form of fractions or ratios."[3]

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3. John H. Walsh, Practical Methods in Arithmetic. (Chicago, D. C. Heath and Co., 1911), p. 274.

Related Research

Arthur Edwards, in a study of percentage, gives the conclusion, "In general, the accomplishments in percentage are disappointing. That in this field, there are genuine opportunities for research calculated to improve our control over learning is sun clear."[4]

Ernest Brown suggests a short-cut method to make the actual mathematical operations, common to the three cases of percentage, as easy as possible. "This method merely takes advantage of the percent sign and the word 'of' and uses their positions in a problem as a clue to the solution."[5]

A comparison of junior high school and elementary school seventh grade pupils by Algae Edwards, in achievement and growth in arithmetic, finds the elementary school group was distinctly superior to the junior high school group in computation, and in growth in problem solving, "there was no difference between the groups."[6]

James Carlos Justice, in an analysis of one hundred eighty-eight seventh, eighth, and ninth grade pupils in arithmetic, reading,

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4. Arthur Edwards, "A Study of Errors in Percentage," National Society for the Study of Education, 29th Yearbook. (1930), p. 640.

5. Ernest N. Brown, "Percent Without Three Cases," School Science and Mathematics, XLIII (March 1943), p. 430.

6. Algae Edwards, A Comparison of Junior High School and Elementary School Seventh Grade Pupils in Achievement and Growth in Arithmetic, Masters 1937, Sam Houston S.T.C. 69 p. ms.

and English, showed that a lack of preparation in tool subjects contributed to a general deficiency in school subjects.[7]

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7. James Carlos Justice, An Analysis of One Hundred Eighty-eight Seventh, Eighth, and Ninth Grade Pupils in Arithmetic, Reading, and English, Masters 1940, Wittenberg. 90 p. ms.

Extent and Scope

The rural, graded, and junior high school pupils in this survey represent a sampling of the Kansas counties of Stafford, Pratt, Reno, and Barton. From the Superintendents of Public Instruction of these counties, school directories, indicating the number of eighth grade pupils and the name of the teacher in each school, were obtained. Tests, accompanied by a personal letter, were sent to these teachers during the first week in December of 1944. Consent was kindly given by the County Superintendents to use their offices as a collection point for the completed tests of their respective county. Of some eleven hundred tests which were distributed, five hundred and sixty-four were administered and returned. These were distributed as shown in the table I, which follows.

TABLE I. DISTRIBUTION OF PUPILS

School	Boys	Girls	Total
Rural	107	122	229
Graded	59	66	125
Junior High	110	100	210
Total	276	288	564

Radium, Kansas
December 1, 1944

Dear Fellow-Educator:

For a master's thesis I plan to measure the retention of percentage of this year's eighth grade students in Pratt, Reno, Stafford, and Pawnee counties.

Because of the expense involved, it is not possible for me to see each of you personally, and it is necessary that I use this method of reaching you. Your County Superintendent has given me a school directory and has indicated the number of eighth graders in your school.

The test is self-administering and requires forty minutes, and, if possible, should be given sometime during the next two or three weeks. No review is to be given for this test and it should be given without any previous announcement.

Your County Superintendent has kindly consented to keep the completed tests at the office and some time when you are going to your local county seat, I hope it is convenient for you to leave the tests with the County Superintendent.

It has been through your kind assistance and cooperation that the success of this work has been possible.

I do appreciate your consideration and --thanks.

Sincerely,

Fred Dellett

Method

In the formation of the percentage test for this study the state adopted texts were examined for type and content of material on the subject of percentage. During the school year 1944-45 the "New Curriculum Arithmetic", by Leo J. Brueckner, grade seven, was used in the rural and the graded schools and "Using Arithmetic Series", grade seven, by Mallory, Cooke, and Loughren, was used in the junior high schools. In these texts it was found that each presented the basic material in percentage as follows:

Processes

1. Changing decimal fractions to percents.

   Example: .12 = 12%

2. Changing percents to decimal fractions.

   Example: 35% = .35

3. Changing percents to common fractions.

   Example: 40% = 2/5

4. Changing common fractions to percents.

   Example: 3/5 = 60%

Cases

1. Finding the part when the percent and the number are given.

   Example: 50% of 20 = 10

2. Finding the number when the part and the percent are given.

   Example: 20% of 50 = 10

3. Finding the percent when the part and the number are given.

Example: 50% of 60 = 30

In order to give the test used in this study of percentage, a degree of validity and objectivity, the Compass Diagnostic Test in Percentage by Ruch, Knight & Studebaker, was examined for content, application, and manner of presentation.

> This test is one of a series of twenty tests, each dealing with some one of the processes or with problem solving. Diagnosis by this particular test is "much more complete and definite than is possible by most diagnostic tests now available".[8]

Suggestions and comments on the resulting test of thirty-one problems were received from teachers who had taught percentage.

The test was compiled in two parts. Part I deals with the four processes. Sixteen problems were used in this section. In the sequence of problems, the simple problems appeared first and the more difficult completed the two sections. Part II contained problems seventeen to thirty-one, inclusive, and dealt with the three type cases of percentage problems. The last six problems of this section were applied problems within the scope and experience of the students. Space for the solution of the exercises of part II was provided, and from this computation, it was possible to determine the particular types of errors made by the pupils in the solution of the problem. The test appears in the appendix.

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8. Brueckner and Melby. Diagnostic and Remedial Teaching. (Chicago, Houghton and Mifflin Co., 1931.) p. 180.

The test papers were collected, graded, and errors or omissions were identified by a code of fourteen parts as explained in the early section of chapter II, following.

CHAPTER II

AN ANALYSIS OF TEST PERFORMANCE

The Frequency Key

In the analysis of the test performance, it is necessary that certain explanations be made in handling the data. These follow: first, is an "Error Identification Code", which is explained below; second, there is found in the chapter, a series of tables, which are explained by use of the code. These techniques were designed to be used in analyzing and classifying the errors made by pupils to whom the test was administered.

Let us describe the "Error Identification Code" and the symbols therein. The code itself consists of a series of letters from the alphabet and symbols used in arithmetic, fourteen in all. Each letter of the alphabet stands for a certain type of error made by the pupils in solving the problems. For example, the letter "D" in the code used in the table, which follows, means that in the solution of any problem the decimal point was misplaced. Again, the letter "M" means that, in the solution of any problem, the student misunderstood as to what was required in solving the problem. Reference to the code identifies the meaning of each character used.

The six tables, which follow, are merely frequency tabulations of the different types of errors made in solving the problems. Explanation of the tables appear after the explanation of the "Error Identification Code".

Error Identification Code

"D" Decimal point misplaced. 32% = 3.2

"M" Misunderstanding or misinterpretation as to what was required of the solution. This error appears frequently in problem 28, where the pupil gives the cost of the material rather than the discount. When asked to change 325% to a mixed number, problem 7, the answer was frequently given as 3.25.

"P" Wrong process used. For example, multiplication was used when the process of division should have been used.

"U" Unaccountable. Answer of the type: 35% = .27.

"C" Computational error. 24 X 3 = 62.

"N" No work shown by which work could be analyzed.

"O" Problem omitted.

"I" Interchange of dividend and divisor in process of division in cases I and II.

"$" Dollar sign omitted.

"%" Failure to add or drop percent sign. $62\frac{1}{2}\% = 5/8\%$

"E" Error in transferring answer from solution to the space provided for same.

"K" Lack of knowledge of the relation of concrete numbers. Some pupils multiplied problems by problems.

"R" Fraction not reduced. 3/8 = 37 2/4%.

"A" Aliquot or fractional part not correctly given. $37\frac{1}{2}\% = \frac{1}{2}$.

Types of Errors for Each Group

Tables II, III, IV, V, VI, and VII, which give the composite picture in the analysis of the errors for the graded school girls, graded school boys, junior high school girls, junior high school boys, rural school girls, and the rural school boys, respectively, are all constructed on the same general plan.

In reading table II, beginning in the upper left hand corner, consecutive numbers above the vertical columns represent the number of the problem, while horizontal rows to the right of the code character give the frequency of this particular error, for which the code character stands for each of the problems.

Let us consider the errors on problem 20, table II. Under problem number 20 and reading across in the row lettered "D", we find 13. This indicates that the graded girls made 13 errors on problem 20 because of misplacing the decimal point. Reading across in the horizontal row from "M" to the number directly under problem 20, we find 0. This indicates that no person misunderstood as to what was required in the solution of this problem. Reading across on the horizontal row identified with "O" to the number directly under problem 20, we find 5. Five (5) omissions were found on problem 20 for the graded school girls. All frequencies of errors are read in this matter; vertical columns give the frequency of the various errors, and the horizontal rows identify the type of error for each particular problem.

The horizontal row of numbers to the right of "Wrong" gives total

of all errors for each problem. The row to the right of "Right" gives the number of correct solutions by the group for each problem, and the last row, "Percent Correct", gives the percent of correct solutions.

TABLE II. FREQUENCY KEY IN TYPE OF ERRORS -- GRADE SCHOOL GIRLS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	2	2	14	14	0	3	3	3	0	2	12	20	10	4	18	8	8	6	3	13	7	12	34	18	17	0	0	2	0	1	1
M	0	0	1	5	7	2	4	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	2	6	1	0	1
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	6	0	2	0	1	1	2	3	2	0	3	1	22	12
U	1	2	4	1	0	1	2	1	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
C	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	12	0	0	1	0	2	0	5	4	3	4	3	2	0	1
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	0	1	4	1	1	10	3	3	2	4	3	1	2
O	0	1	3	2	3	3	4	1	0	0	0	0	0	0	0	0	0	1	2	5	4	2	1	6	6	0	1	0	1	2	9
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	10	1	1	0	0	6	4	0	0	0	0	0	1
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	3	0	1	0	1	0
K	1	3	8	13	7	7	9	7	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	4	0	2	1	0
R	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	1	6	3	0	0	6	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Wrong	6	16	33	35	17	23	23	15	0	2	12	23	10	4	19	8	23	19	15	23	16	18	37	47	37	22	13	19	11	29	28
Right	60	50	33	31	49	43	43	51	66	64	54	43	56	62	47	58	43	47	51	43	50	48	29	19	29	44	53	47	55	37	38
Percent Correct	91	76	50	47	74	65	65	77	100	97	82	65	85	94	17	88	66	71	77	65	76	73	44	20	44	67	79	71	83	56	58

Referring to Table II for the grade school girls, we find 237 errors, 39% of the 604 total errors, were due to a misplaced decimal point. Thirty-four of these errors were found on problem 23, ".03% of .16 = .000048". This represents an uncommon decimal fraction and the solutions indicate that, because it involves such a number, the pupils were not certain of the proper position of the decimal point. Such an assumption is further confirmed, when we note that the second highest number of errors of this type appears in problem 12. Here a large percent is found, "30 = 3000%".

These facts show there is a definite lack of application or misinterpretation of the principle of percent. Ten errors on problem 26 were because of misunderstanding as to what was required of the solution. These ten errors do not indicate a lack of knowledge of percentage and is irrelevant to this study.

Problem 30 has twenty-two errors for the group, grade school girls, which show that division should have been used instead of multiplication in the solution.

Twelve errors on problem 17 were due to computational errors. The multiplication of the factors 14 and 76 authenticated this assumption. Some students gave the product as 9.64 instead of 10.64.

Ten pupils in the process of division interchanged the dividend and the divisor in solving problem 19, "2 = 2% of 100." Problem number 4 showed that thirteen lacked a knowledge in changing twelve to a percent. On the whole, prevalency of the misplacing of the decimal point,

the author believes, increased the frequency of errors of types other than those of placing the decimal point.

TABLE XIII. FREQUENCY DISTRIBUTION OF ERRORS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	0	2	12	9	0	4	2	3	1	3	13	20	7	8	13	9	3	12	5	8	14	15	30	9	17	0	2	1	0	1	1
M	0	0	0	3	8	6	7	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	2	3	1	0	1
P	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	8	0	12	1	3	6	9	7	4	2	8	3	15	9
U	0	4	7	5	1	3	2	5	0	1	1	1	0	0	0	0	0	1	0	0	1	1	0	0	0	1	0	1	0	0	1
C	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	1	0	5	0	1	2	2	3	2	3	0	1	0	0
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	3	5	2	3	2	5	6	6	3	3	7	3	8
O	0	0	0	0	4	2	5	2	0	0	0	0	0	0	0	0	1	0	0	3	1	2	0	7	7	1	2	1	1	1	6
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	22	1	1	0	0	13	5	0	1	0	1	2	1
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	0	1	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
E	0	0	0	0	0	0	0	0	1	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	1	0	0	0
K	2	2	9	13	6	8	10	7	0	0	1	0	0	0	0	0	2	1	1	1	0	0	1	0	0	0	6	1	4	2	1
R	0	0	0	0	0	1	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	2	4	3	0	0	6	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Wrong	5	13	31	30	19	30	31	27	2	4	16	21	7	9	13	9	16	26	13	35	20	25	44	45	45	22	21	21	18	25	28
Right	54	46	28	29	40	29	28	32	57	55	43	38	52	50	46	50	43	33	28	24	39	34	18	14	14	37	38	38	41	34	31
Percent Correct	92	78	48	49	68	49	48	54	97	93	73	64	87	85	78	85	73	56	48	41	66	58	31	24	24	63	64	64	70	58	53

Table III shows that thirty and twenty pupils had difficulty in properly locating the decimal point on problems 23 and 12, respectively. Errors of this type were the more numerous with the grade school girls. Two hundred twenty-four, 31% of the 686 total errors, were of a decimal point position nature.

Solutions for problems 30 and 20 showed 15 and 12 errors, respectively, because the wrong process was used.

Twenty-two and thirteen errors on problems 19 and 24, respectively, were due to interchange of parts in division. Problem 4, "12 = 1200%," showed a lack of fundamental understanding. This group also showed a definite deficiency of knowledge in the placing of the decimal point.

TABLE IV. FREQUENCY KEY IN TYPE OF ERRORS -- -- JUNIOR HIGH SCHOOL GIRLS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	0	2	31	23	3	2	4	2	1	7	28	47	17	13	35	19	13	36	15	13	19	28	56	25	23	2	2	5	0	3	2
M	0	0	0	21	8	19	9	18	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	18	9	7	1	3	1
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	14	7	21	4	4	7	11	15	1	6	6	2	42	25
U	0	3	13	5	0	8	2	6	0	0	0	0	1	0	0	0	0	0	0	0	3	0	0	0	1	0	0	0	0	1	1
C	3	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	3	1	1	2	0	5	0	4	2	4	7	3	6	1	2
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	3	3	3	5	7	1	3	4	1	7	4	6
O	0	3	7	8	8	13	11	8	0	1	0	2	0	1	0	1	4	8	4	27	13	16	8	34	28	1	2	0	4	5	13
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	35	5	0	0	0	4	4	0	2	0	3	3	4
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	1	0	1	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	1	0	0
E	0	0	3	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
K	2	3	12	14	31	4	19	5	0	0	0	0	0	0	1	0	1	0	2	0	0	0	0	0	0	0	5	0	9	0	0
R	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	3	10	0	0	0	8	1	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
Wrong	8	21	66	71	50	55	49	45	3	8	28	49	18	14	36	20	29	65	66	71	42	57	76	87	75	31	37	23	34	63	54
Right	92	79	34	29	50	45	51	55	97	92	72	51	82	86	64	80	71	35	34	29	58	43	24	13	25	69	63	77	66	37	46
Percent Correct	92	79	34	29	50	45	51	55	97	92	72	51	82	86	64	80	71	35	34	29	58	43	24	13	25	69	63	77	66	37	46

Four hundred seventy-six or 35% of the 1351 total errors of the junior high school girls (Table IV) were of a decimal point nature. Problems 23 and 12 had 56 and 47 errors, respectively. These examples represent small decimals and large percents. The process used accounted for 171 errors on the test papers for this group. Omissions were prevalent and 230 were tabulated. No attempt on the solution was made in these problems, and one is not at liberty to ascertain the difficulty encountered because several factors enter; for instance; knowledge of fundamentals, and lack of time.

One hundred eight junior high school girls showed a lack of knowledge in the handling of percentage problems. Again, placing of the decimal point was the predominant type of error.

TABLE V. FREQUENCY KEY IN TYPE OF ERRORS - - JUNIOR HIGH SCHOOL BOYS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	0	1	30	25	4	3	5	3	2	8	20	57	21	11	29	14	9	28	11	19	21	24	67	13	20	2	2	6	1	1	8
M	0	1	2	20	15	14	14	13	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	13	7	7	3	1	0
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	8	20	5	21	4	5	3	15	17	3	3	6	1	29	22
U	0	6	12	10	3	5	7	4	1	2	1	2	1	1	0	1	1	1	1	1	4	0	1	0	1	0	0	0	2	1	0
C	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	8	1	1	3	0	8	3	3	4	7	8	2	10	0	0
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	8	4	11	2	4	5	16	10	4	11	3	12	8	5
O	0	6	9	8	8	15	13	10	1	1	3	2	1	1	1	2	6	11	5	24	14	23	11	38	35	1	3	1	3	5	10
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	39	6	1	0	0	12	6	1	2	0	2	0	4
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	4	0	2	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	1	0	0	0	1	0	0	0	1
E	0	1	0	0	0	0	0	0	1	0	0	0	0	1	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0
K	0	4	13	21	32	3	12	2	0	1	1	0	0	0	1	0	0	1	2	0	0	1	1	1	0	1	2	0	5	4	3
R	0	0	0	0	0	2	5	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
A	0	21	0	0	0	11	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
Wrong	0	40	67	84	62	53	56	36	5	13	26	62	23	14	31	17	39	76	69	85	46	66	83	98	93	34	41	29	39	51	53
Right	110	70	43	26	48	57	54	74	105	97	84	48	87	96	79	93	71	34	41	25	64	44	17	12	17	76	69	81	71	59	57
Percent Correct	100	64	39	24	44	52	49	67	95	88	76	44	79	87	72	85	65	31	37	23	58	40	15	11	15	69	63	74	65	54	52

Here, in the examination of the errors made by the junior high school boys (Table V) we again note that problems 23 and 12 showed the greatest frequency in incorrect position of the decimal point. Four hundred sixty-five of the total 1501 errors was due to inaccurate handling of the decimal point. This represents 31% of the total errors. The solutions of problems 23 and 12 showed 67 and 57 errors of this nature, respectively. These are the same two problems which, likewise, showed the greatest number of errors due to decimal point placement in Tables II, III, and IV. Use of the wrong process, division or multiplication, accounted for 164 or 11% of the errors.

Omissions, on problems 17 to 25, inclusive, totaled 167. These examples represent applied problems in percentage and each is identified with one of the three cases of percentage.

Thirty-nine pupils interchanged the divisor and the dividend on problem 19. This represents the same general trend for this example with all the groups. This problem, "2 = $\underline{2\%}$ of 100", involves the process of division but multiplication was used.

Sixty-two of the one hundred ten pupils could not change "1% to its fractional equivalent." This deplorable circumstance should challenge our thinking. This problem, alone, warrants the conclusion that percentage is being poorly taught.

TABLE VI. FREQUENCY KEY IN TYPE OF ERRORS - - RURAL SCHOOL GIRLS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	0	3	35	20	4	6	5	6	2	17	44	63	27	10	48	25	13	25	16	24	27	34	68	39	50	2	5	2	3	3	5
M	0	0	0	10	7	13	10	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	3	3	0	0	0
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	12	2	11	4	3	6	5	5	9	1	21	4	46	14
U	1	7	8	10	0	9	2	9	0	0	0	2	0	0	0	0	1	0	0	0	2	0	1	0	1	3	2	3	0	1	0
C	0	4	1	0	0	1	1	1	0	0	0	0	0	0	0	0	10	2	0	6	0	2	2	9	2	4	9	2	7	0	3
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	6	2	6	9	4	3	15	7	10	12	6	18	11	12
O	0	2	4	4	11	15	12	13	2	3	2	2	1	0	2	1	1	5	3	15	10	4	6	18	12	5	7	10	6	7	22
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	32	5	4	0	0	18	14	0	5	0	2	0	2
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	10	0	7	0	8	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	0	2
E	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0
K	2	5	16	34	18	22	25	25	0	0	0	2	0	0	1	0	0	1	1	0	0	1	1	0	1	6	12	1	9	6	7
R	0	0	0	0	0	1	4	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
A	2	10	0	0	0	8	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Wrong	5	31	65	78	40	75	59	67	5	20	46	69	28	10	51	26	32	63	56	68	56	49	87	104	93	56	56	56	50	82	67
Right	117	91	57	44	82	47	63	55	117	102	76	53	94	112	71	96	90	59	66	54	66	73	35	18	29	66	66	66	72	40	55
Percent Correct	94	75	47	35	67	39	52	42	94	84	62	44	77	92	58	79	74	48	54	44	54	60	29	15	24	54	54	54	59	33	45

Six hundred thirty-one of the 1650 errors and omissions made by the rural school girls (Table VI) were due to misplacing the decimal point. This represents 38% of all the errors and omissions. Problems 23 and 12 showed the greatest frequency of this type of error, 68 and 63 errors, respectively. Large percents and small decimals baffled the pupils.

Forty-six pupils multiplied when they should have divided in the solution of problem 30. Omissions totaled 205 for the group on the test.

Thirty-two pupils interchanged the dividend and the divisor in solving problem 19, "2 = 2% of 100". Only 67% of the students solved problem 5, "1% = 1/100". Results on the last two mentioned problems definitely show a lack of mastery.

TABLE VII. FREQUENCY IN TYPE OF ERRORS - - RURAL SCHOOL BOYS

Error	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
D	1	8	38	19	3	7	6	7	2	11	43	54	21	18	42	24	14	34	15	13	25	26	62	22	35	2	7	9	2	4	10
M	0	0	0	12	13	10	11	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	1	3	1	0	1
P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	13	0	20	5	1	5	9	7	9	1	16	3	23	18
U	1	2	14	8	1	6	6	3	1	1	1	0	1	1	1	2	0	1	1	1	3	0	0	0	0	1	2	1	1	0	0
C	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	12	1	0	3	2	8	3	4	3	4	7	2	1	1	3
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	5	2	2	9	13	9	4	20	15	11	18	11	20	10	16
O	1	6	7	6	10	21	17	18	1	0	0	1	0	0	0	0	0	5	3	17	9	8	10	19	19	1	6	1	6	11	12
I	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	7	35	7	1	0	0	13	10	0	3	0	1	2	1
$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	3	0	4	0
%	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	2	1	0	0	0	0	0	1	0	0	0	0	0	0	0
E	0	0	0	0	0	0	1	0	2	0	0	0	0	0	0	0	1	0	0	0	0	1	2	0	0	1	0	3	1	1	2
K	1	7	12	31	22	14	23	13	0	0	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	2	15	3	9	4	4
R	0	0	0	0	0	4	5	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	2	21	0	2	0	4	0	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
Wrong	6	44	71	78	49	66	69	59	6	12	44	56	22	19	44	26	36	64	57	70	58	53	86	88	89	43	62	52	45	60	67
Right	101	63	36	29	58	41	38	48	101	95	63	51	85	88	63	81	71	43	50	37	49	54	21	19	18	64	45	55	62	47	40
Percent Correct	94	59	34	27	54	38	35	45	94	87	59	48	79	82	59	75	66	40	47	35	46	50	20	18	17	60	42	51	58	44	37

In Table VII, five hundred eighty-four of the 1601 errors and omissions by the rural school boys was due to misplacing the decimal point. This accounted for 36% of the total errors. Problems 23 and 12 showed 62 and 54 errors, respectively. Pupils encountered difficulty with large percents and small decimal fractions on these two problems.

Problem 30 showed the most numerous errors were due to process. For this groups 205 omissions on the test were tabulated. Thirty-five pupils experienced difficulty with problem 19 by interchanging the dividend and the divisor.

Attention is called to the fact that in the examination of the prevalent types of errors for all the groups, we note the correspondence of frequencies of errors due to decimal point on problems 12 and 23, and the interchange of dividend and the divisor in problem 19.

Problem 5, "1% = $\frac{1}{100}$", shows correct solutions for 59.5% of the pupils for the groups, problem 19, "2 = 2% of 100", gives an average of 49.5%. Problem 12, "30 = $\frac{3000}{\%}$," is correct on 52.7% of the papers. Only 27.2% of the pupils participating in this test solved problem 23, ".03% of .16 = .000048."

Review of these results definitely moves one to form the conclusion --percentage is being poorly taught.

Analysis of Errors for Each Problem

In the discussion which follows, the more numerous errors made in solving each particular problem will be analyzed. The percent of problems solved correctly by the grade school girls, grade school boys, junior high school boys, rural school girls, and rural school boys will be given for each problem discussed, and in the order named. Problems five to eight, inclusive, will be discussed concurrently, since they represent the process of changing percents to fractions or mixed numbers. Problems nine to sixteen, inclusive, involve the moving of the decimal point and the adding or dropping of the percent sign. These are covered in the latter part of Chapter II.

Part I of the Test

Problem 1. "$\frac{1}{4} = 25\%$."

The solutions of problem 1 showed that 90.0% of the grade school girls gave a correct solution. The tabulations showed that two had difficulty in placing the decimal point. Because this problem represents a very common aliquot part one would be led to say that but little computation was used. This fact is verified on the test papers. Ninety-two percent of the boys of this type of school organization gave correct solutions. Ninety-two percent of the junior high school girls gave correct solutions. All 110 boys of the junior high school showed complete mastery. Ninety-four percent of the rural school girls and 94.4% of the rural school boys presented correct solutions.

Problem 2. "5/8 = $62\frac{1}{2}\%$."

This problem had less correct solutions, than were shown on the previous example, because this aliquot part was not as common. The grade school girls showed 16 scattered errors and made a score of 75.8%.

The number of errors of the grade school boys was 13, of which there were four unaccountable and 4 mistakes in recalling aliquot parts. A score of 78% was made.

Ten aliquot parts were wrong in 21 total mistakes in the solutions of the junior high school girls. Seventy-nine percent of the problems were correct.

Twenty-one problems lacked the proper aliquot part in the solutions of the junior high school boys, and a score of 63.6% was made.

Aliquot parts bothered 10 of the rural school girls who made 31 errors. The percent correct was 74.6%.

Twenty-one rural school boys missed because of aliquot parts when 44 errors were made in all. The score was 58.9%.

Problem 3. "$4\frac{1}{2}$ = 450%

The most common error in each group was due to place value. The grade school girls had 14 errors of this type in 33 total errors; the grade school boys, 12 in 31; the junior high school girls, 31 in 66; junior high school boys, 30 in 67; the rural school girls, 35 in 65; and the rural school boys, 38 in 71. The percents of problems correct were 50, 47.5, 34, 39.1, 46.7, and 33.6, respectively.

Problem 4. 12 = 1200%.

A typographical error occurred in this example when 1/12 was written as 12. A repetition appears here since problem 12, "30 = 3000%," is of the same type. The scores of problems solved correctly were: grade school girls, 47.0%; grade school boys, 49.2%; junior high school girls, 34%; junior high school boys, 23.6%; rural school girls, 36.4%; and rural school boys, 27.1%.

Problems 5 to 8, inclusive

5. 1% = 1/100
6. 325% = $3\frac{1}{4}$
7. $37\frac{1}{2}\%$ = 3/8
8. $12\frac{1}{2}\%$ = 1/8

The process of changing percents to fractions or mixed numbers is given here.

The percent of mastery is as follows: grade school girls, 70.5%; grade school boys, 54.7%; junior high school girls, 50.3%; junior high school boys, 53.0%; rural school girls, 50.6%; and rural school boys, 43.2%.

Problems 9 to 16, inclusive

9. .35 = 35%
10. 3.06 = 306%
11. .0026 = .26%
12. 30 = 3000%
13. 4.5% = .045
14. 32% = .32
15. .003% = .0003
16. 521.6% = 5.216

TABLE VIII. COMPREHENSIVE ANALYSIS OF PROBLEMS 9-16

Group	Decimal point errors	Total of all errors	Percent of total errors	Percent correct
Grade school girls	74	78	94.9	85.2
Grade school boys	74	81	91.4	82.8
Junior high school girls	167	176	94.9	78.0
Junior high school boys	162	188	86.2	78.6
Rural school girls	226	245	92.2	74.9
Rural school boys	215	229	93.9	73.2

Reference to Table VIII indicates that in the solutions of problems 9 to 16, inclusive, the grade school girls misplaced the decimal point in 74 cases and, in all, 78 errors were made. Errors in placing the decimal point for the grade school girls represented 94.9% of all errors and the percent of problems solved correctly was 85.2%.

Statistics for the other groups are found in the same manner by referring to the above table.

From the table it can be seen that the greater part of the difficulties of these problems embodies the direction of moving the decimal point and the number of places.

Part II of the Test

Problems 17 to 31, inclusive, represent the three cases of percentage problems. Problems 17 to 25, inclusive, are set up in the form, percent times base equals part. In omitting either the base, part, or the percent, one of the three cases of percentage is formed. In the applied problems 26 to 31, inclusive, it is necessary that the pupil set up the relationship of the given quantities and then solve the case formed.

Problem 17. 14% of 76 = 10.64

Eight grade school girls misplaced the decimal point and twelve had errors in computation in the process of multiplication. Sixty-five and two-tenths percent presented correct solutions.

Sixteen errors were made by the 59 grade school boys, five because of process and four because of computational errors. Seventy-two and nine-tenths percent solved the problem correctly.

Twenty-nine, of the one hundred junior high school girls, made errors. Of these, thirteen were due to misplacing the decimal point and six used the incorrect process. Seventy-one percent mastered the problem.

Errors of the junior high school boys were scattered. Nine misplaced decimal points, eight incorrect processes, and eight computational errors were the more numerous. Mastery was shown by 64.5%

Errors of the rural school girls were primarily of two types: 13 misplaced decimal points and 10 computational errors. This gave a

percent solved of 73.9%.

The 107 rural school boys showed 14 misplaced decimal points and 12 computational errors. Absence of difficulty was shown by 66.4%.

Problem 18. 7% of $\underline{600}$ = 42

The grade school girls made 6 errors each on decimal point and process. Correct solutions were shown in 71.2% of the studies.

Twelve errors in decimal point and eight in process were shown by the grade school boys to give a percent of 55.9%.

The junior high school girls made 36 errors in decimal point placement and 14 errors in process to give a percent of problems correct of only 35%.

Only 30.9 percent of the junior high school boys solved this problem. Twenty-eight errors were due to decimal point and 20 due to error in selection of process.

Twenty-five inaccuracies of the rural school girls were due to decimal point placement. Correct solutions were given on 48.4% of the papers.

Inaccuracies due to a misplaced decimal point were predominant with the rural school boys and 40.2% of the solutions were correct.

Problem 19. 2 = $\underline{2\%}$ of 100

Ten division processes showing an interchange of parts represented the greater part of the faults when 77% of the grade school girls solved the problem.

Twenty-two of the thirty-one errors of the boys from the grade schools were due to interchange of parts in division of the problems while 40.7% were perfect.

In 100 cases of junior high school girls, 15 showed fallacies in placing the decimal point and 35 interchanged parts when dividing. Of the solutions, 34% were without error.

Thirty-nine junior high boys interchanged the dividend and the divisor, which left 37.3% of the problems correct.

Interchange of parts in division was shown by 32 rural school girls and 10 misplaced the decimal point when 56 errors were made. Exactness was shown in 54.1% of the cases.

Interchange of parts meant a loss to 35 rural boys, when, at the same time, 15 errors were due to decimal point. Correct solutions were given by 46.7% of the participants.

Problem 20. $420 = 105\%$ of $\underline{400}$

In this problem, where the base was required, it was necessary that the work involve percent and it was shown by the grade school girls that 13 of the 23 errors were due to placing of the decimal point. Correct work was shown by 65.2%. The majority of the errors arose from placing of the decimal point.

Grade school boys had 35 errors in 59 cases. Eight had difficulty with the decimal point. Twelve used the wrong process. The score on this exercise was 40.6%.

Thirteen errors of the junior high school girls were of a deci-

mal point nature while 27 omitted the problem and 21 used the wrong process. Seventy-one errors in 100 cases were tabulated which gave a percent of twenty-nine for the correct solutions.

Only 22.7% of the junior high school boys had this problem correct, while in the 85 errors, 19 were due to decimal point, 21 used the wrong process and 24 omitted the problem.

Rural school girls misplaced the point in 24 cases, selected the wrong process in 11 cases, and 15 solutions were omitted. The percent of problems correct was 44.3%.

Thirty-four and six-tenths percent of the grade school boys solved this problem correctly. Twenty used the wrong process and 17 omitted the example.

Problem 21. $\underline{10\%}$ of 85 = 8.5

All the groups experienced less difficulty on this example than on the previous. Of the 66 grade school girls, seven misplaced the decimal point, while 75.8% of the solutions were correct.

Fourteen of the 20 errors of the grade school boys were made by misplacing the decimal point. Mastery was shown in 67.8% of the cases.

Nineteen junior high school girls improperly placed the decimal point and 13 omitted the example. Fifty-eight percent of the examples were correct.

The errors of the junior high school boys were comparable to those of the junior high school girls. Twenty-one had difficulty with

the decimal point and 14 individuals omitted the example. Fifty-eight and two-tenths percent mastered the situation.

Twenty-seven rural school girls missed the place value and 10 omitted the example. Correct solutions were 54.1% of the 122 cases.

The decimal point was misplaced in 25 cases with the rural school boys and thirteen errors could not be diagnosed because no work was shown. Forty-five and eight-tenths percent of the problems were correct.

Problem 22. 119.5% of 2000 = 2390

The greatest number of errors in each division is attributed to placing of the decimal point. We are here working with a problem involving a percent greater than one hundred percent.

Of the 8 errors made by the grade school girls, 12 were due to place value. Seventy-two and seven-tenths percent suffered no loss.

In misplacing the decimal point in 15 instances the grade school boys came through with 57.3% of the problems correct.

Of the 57 errors by the junior high school girls, 28 were the result of decimal point placement and 16 were omissions. Forty-three percent of the solutions were correct.

Forty-seven misplaced decimal points and omissions were found in the work of the junior high school boys. Forty percent mastered the occasion.

Thirty-four of the 49 errors by the rural school girls were attributed to a misplaced decimal point. This group rated 59.8%.

Twenty-six boys of the rural school misplaced the decimal point making 50.5% of the solutions correct.

Problem 23. .03 % of .16 = .000048.

Thirty-four of the thirty-seven errors by the grade school girls were due to misplacing of the decimal point. Their score was 43.9%.

With the grade school boys the decimal point was incorrectly placed in 30 of the 41 cases. Their score was 30.5%.

The junior high school girls misplaced the decimal point in 56 of the 76 errors. Only 24% of the problems were correct.

With the junior high school boys, errors in placing the decimal point predominated when 67 of the 93 errors were of this nature. Only 15.5% of the examples were correct.

Of the 87 errors of the rural school girls, 68 were because of a misplaced decimal point. The score was 29.1%.

Of 86 errors by the rural school boys, 62 were attributed to place value. Nineteen and six-tenths percent of the examples were correct.

Problem 24. .325% of 461 7/13 = 1.5

Eighteen misplaced decimal points and ten problems with no solution shown, were the most numerous type when the grade school girls made 47 errors. The score was 28.8%.

The grade school boys had a score of 23.7% on this example. Nine errors each were found in decimal points and process while 13 errors were due to interchange of parts in division. Forty-five errors

in all were found.

Junior high school girls had difficulty with place value in 25 instances and thirty-four problems were omitted. Eighty-seven errors were identified in all and 13% of the problems were correct.

Of the ninety-eight errors made by the junior high school boys, 15 were of process, 16 had no work shown, and 38 were omissions. The score was 10.9%.

The grade school girls showed 39 errors because of decimal point in a total of 104 mistakes. Fourteen and eight-tenths percent of the examples were correct.

Twenty-two decimal point errors, twenty errors with no solutions, and nineteen omissions, in a total of 88 mistakes, gave the junior high school boys a score of 17.8%.

Problem 25. .12% of $\underline{13\ 1/3}$ = .016

The placing of the decimal point represents the greatest number of errors in each group, with the exception of the junior high school girls and boys, where we find more omissions than decimal point errors.

Seventeen of the forty-five errors of the grade school boys were due to the decimal point. The score was 23.7%.

Seventeen of the twenty-eight errors were of a place value with the grade school girls. Forty-two and four-tenths percent gave correct solutions.

The junior high school girls misplaced the decimal point in 23 cases and there were 28 omissions in the 75 errors. Twenty-five per-

cent of the examples were correct.

The junior high school boys made 20 errors in decimal point and there were 35 omissions. The score was 15.5%.

Of 93 errors of the rural school girls, 50 were because of decimal point placement. Mastery was shown in 23.7% of the cases.

The rural school boys made a score of 16.8% when 35 of the 89 errors were of a place value nature.

Problem 26. How much is saved by purchasing an article listed at $5.20 at 25% off? $1.30

Here the girls of grade school had difficulty in that 10 of the 22 errors tabulated were due to a misinterpretation of what was to be found. These answers, which were incorrect, gave the selling price rather than the amount saved. Sixty-six and six-tenths of the solutions were correct.

The grade school boys had 6 errors because of misinterpretation and 6 problems had no solution by which the error could be diagnosed. In the solution, six boys used the wrong process. Of the 59 cases, 37 were correct, giving a percent of 62.7.

The junior high school girls had 18 cases of misinterpretation of what was required--the selling price was given instead of the discount. Correct solutions represented 69%.

Of the thirty-four errors made by the junior high school boys, thirteen were attributed to misinterpretation and seven were computational errors. Sixty-nine percent had correct solutions.

Of the 56 errors made by rural school girls, seven were due to misinterpretation, nine involved the wrong process, ten showed no work and ten carelessly omitted the dollar sign. Fifty-four and nine-tenths percent gave correct solutions.

Fifty-nine and eight-tenths percent of the rural school boys gave correct solutions. Predominant errors were: nine due to process and eleven with no solution.

Problem 27. Herbert made 15 free throws out of 24 attempts. What percent of the "tries" were successful? $62\frac{1}{2}\%$

Eighty and three-tenths percent of the grade school girls solved this problem, which represented case III, where the percent was to be found. Four errors, each, of computation and lack of knowledge were found.

Lack of knowledge accounted for six of the twenty-one errors with the grade school boys, which gave a grade of 64.4%.

Nine junior high school girls gave the percent of throws unsuccessful, six used the wrong process and seven made computational errors. Sixty-three percent showed mastery.

Seven junior high school boys misinterpreted as to what was required. Eight made computational errors, and in eleven cases no work was shown. Sixty-two and seven-tenths of the solutions were correct.

Nine rural school girls showed a computation error, twelve had no work shown, seven omitted the example and twelve lacked knowledge of this particular problem. Fifty-four and one-tenth percent gave

correct solutions.

Eighteen rural school boys showed no solution, although, an answer was given. Fifteen definitely lacked knowledge. Forty-two and one-tenth percent were correct.

Problem 28. Mr. Allen bought $36.00 worth of lumber at his summer camp. He received a discount of 3% for cash. What was his saving by paying cash? $1.08

Six grade school girls misinterpretated as to what was required in this problem involving case I. Four errors had no work shown. Seventy-one and two-tenths percent comprehended this problem.

Eight grade school boys used the wrong process. The score was 64.4%.

Seventy-seven percent of the junior high school girls had correct solutions. The greatest number of errors shown was seven in misinterpretation.

Six junior high school boys misplaced the decimal point, seven misinterpreted and six used the wrong process. Seventy-three and six-tenths percent mastered the problem.

Twenty-one rural school girls used the wrong process while the group made a score of 54.1%.

Of the rural school boys, nine misplaced the decimal point, sixteen used the wrong process, and in eleven cases no work was shown. Only 51.4% mastered this problem.

Problem 29. Harry had 24 examples correct in a test of 25 problems. What percent of the examples did he have correct? 96%

Less errors were shown here when 83.3% of the grade school girls solved the problem correctly.

Seven grade school boys made errors which showed no work and the group made a score of 69.5%.

Seven incorrect problems for the junior high school girls showed no solution while the group made a grade of 66%.

Sixty-four and five-tenths percent of the junior high school boys solved the problem correctly and 10 computational errors and 12 errors with no solution shown were tabulated.

The rural school girls had 18 incorrect answers with no solution and 9 represented a lack of knowledge. Fifty-nine percent gave the correct solution.

Twenty errors of the rural school boys had no solution while nine lacked a knowledge of percent. The score was 67.3%.

Problem 30. During bargain week Charles saved $1.60 by purchasing a rifle at a discount of 20%. For how much was the rifle originally priced? $8.00

Here the grade school girls showed that the decisive trend of errors was due to the use of the wrong process. Twenty-two of the twenty-nine errors were of this nature. Fifty-six and one-tenth of the girls solved the problem correctly.

Fifteen or 60% of errors made by the grade school boys were due to misuse of process. Only 57.6% of the boys solved this problem.

Forty-two of the 63 errors made by the junior high school girls, was due to misuse of process. Only 37% of the girls solved this problem.

Twenty-nine junior high school boys had difficulty in selecting the proper process. Fifty-three and six-tenths percent of the solutions were correct.

The rural school girls showed the lowest grade of all, 32.8%. Forty-six of the eighty-two errors were due to use of the wrong process.

Twenty-three rural school boys were bothered with use of the incorrect process and 43.9% solved the problem.

This problem represents the highest mortality due to use of the wrong process. The boys show the better scores and it may have been because of interest of the rifle.

Problem 31. In Lincoln school there were 291 pupils present. If this represents 97% of the enrollment, what is the enrollment? 300

Twelve of the twenty-eight errors of the grade school was due to incorrect choice of process and nine omitted this example. Fifty-seven and six-tenths percent of the grade school girls solved the problem.

Nine of the grade school boys used the improper process and eight had no work shown to give a percent of 52.5% of the problems correct.

Twenty-five junior high school girls used the wrong process while 13 omitted this example. Forty-six percent solved the problem.

Twenty-two junior high school boys used the wrong process while ten omitted the example. Correct solutions were shown by 51.8%.

In making a percent of forty-five and one-tenth, the rural

school girls showed fourteen errors due to process and twenty-two omissions.

The rural school boys made a score of thirty-seven and four tenths when ten errors were due to decimal point, eighteen because of process, no work was shown in sixteen errors and there were twelve omissions.

CHAPTER III

GENERALIZATIONS, RECOMMENDATIONS AND SUGGESTIONS

Generalizations and Recommendations

As one thoughtfully reflects upon the performance of elementary pupils in solving the problems in percentage used in this investigation, it appears that a few general recommendations pertaining to the teaching and the presentation of percentage should follow. However, there are those teachers of percentage who might question the validity of some of the recommendations when one recalls the words of Thorndike, "The subject of system and organization in arithmetic is too broad and intricate to be summed up in a brief way".[9]

First, many errors in solving the problems were due to carelessness on the part of the pupils in number notation. "Children should be taught to make legible figures and to space them properly. The commonest error is to write them too close together and to make the fractions too small."[10] Incorrect solutions were also due to the fact that some pupils were inaccurate in the copying or the transferring of numbers from the statement of the problem into the mechanics of

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9. Thorndike, op. cit., p. 102.

10. Thorndike, op. cit., p. 17.

solving the problem. Occasionally, a problem was worked correctly, but in transfer of the answer to the space provided, an error developed.

Second, an occasional student paid no attention to the fundamental process used with concrete numbers. Dollars and percents were added, and problems were multiplied by problems. Here, a sense of relationship of numbers need be established. "In order to make a diagnosis of the causes of a pupil's deficiency in arithmetic, the teacher should have a grasp of the skills that constitute the process, an appreciation of how complex the process appears to the learner, and a knowledge of the most common causes and kinds of difficulties revealed by the study of the work of the pupils in the grade."[11]

Third, a great number of errors could have been eliminated had the students made an estimate of the answer. "Always estimate the answer in advance, write down the estimate, and compare it with the result."[12]

Fourth, pupils are aware of the "two place" change of the decimal point but some are lost as to knowing whether it should be moved to the right or the left. Percentage means by the hundredths, thus: .23 is equivalent to 23% and vice versa. By visualizing this or a

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11. Brueckner and Melby. Diagnostic and Remedial Teaching. (New York, Houghton and Mifflin Co. 1931), p. 190.

12. John Dickey. "The Value of Estimating Answers to Arithmetic Problems and Examples". Elementary School Journal, XXXV (Sept. 1934)

related statement, the student should be able to establish the relationship between hundredths and percent, and to properly shift the decimal point. In some of the cases, if the answer was a small decimal fraction or a large percent, the students were a bit dubious of the resulting answer.

Fifth, examination of a few of the solutions given after fractions were changed to percents revealed that where the students' recollection of aliquot parts had failed him, he was hopelessly lost as to what to do. Textbooks teach that a ratio is a quotient and may be expressed in fractional form. From this statement, we see that a fraction is a quotient, the numerator is the dividend, and the denominator of the fraction, the divisor. Applying this rule, 5/8 means 5 divided by 8. The solution gives an abstract number which may then be changed to percent.

Sixth, let us consider the process of changing a percent to a fraction. Percent is another name for hundredths.

$37\frac{1}{2}\% = .37\frac{1}{2} = 37\frac{1}{2}/100$

Applying the principle, a fraction is a quotient, we have:

$37\frac{1}{2} \div 100 =$

$75/2 \div 100 =$

$75/2 \times 1/100 = 3/8$

Seventh, let us direct our attention to those errors common to the solution of the problems representing the three cases of percent. In these cases, we have the relation:

percent X number = part

20% X 600 = 120

.20 X 600 = 120

Here we have the two factors which, when multiplied together, give the product, and either factor may be found by dividing the product by the known factor. When we have the two factors, naturally we multiply, .20 X 600 = 120. Should we be asked to find a missing factor, .20 X ? = 120, 120 ÷ .20 = 600 (Product divided by known factor gives the missing factor). No attempt should be made to introduce literal equations in the three cases of percent. Better still, a more common situation, 2 X 4 = 8, may be used to instill the relationship of the parts.

"Experienced people sometimes have difficulty in determining which number is the divisor, and not few teachers; in their own work, employ the old rule: Divide the 'is' by the 'of' . . .".[13] "A rule should grow out of a pupil's experience and sum up what he has already learned or is learning to do, in the most helpful way for memory and future guidance."[14]

In further diagnosis of the errors, the author found instances where the student divided the "part" by the "number" and then multiplied by 100 to get the percent. This would be considered as use of

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13. John H. Walsh. Practical Methods in Arithmetic (Chicago, D. C. Heath and Co. 1911), p. 267.

14. Thorndike, op. cit. p. 229.

a memorized rule which eliminates reasoning, and, in some instances, results in confusion. "Under right conditions, basing the cases on the 'product and two factors principle' brings most excellent results. The pre-eminent difficulty lies in the making the matter clear to the children. If one small part is left vague, the pupil will at once commence to depend upon his memory and not upon his thought powers. As a final business mode of solution, the 'product and two factors' plan is probably superior to all others."[15] The prevalence of shifting the decimal point in the divisor and dividend a sufficient number of places to make the divisor a whole number was a common practice among the students. In such a practice, each figure loses its true place value, and, in no case, should be tolerated.

Suggestions

In conclusion, when we consider the scores of the grade school girls, grade school boys, junior high school girls, junior high school boys, rural school girls, and rural school boys as being 70.5%, 62.5%, 56.5%, 56.0%, 56.4%, and 51.7%, respectively, on a representative test in percentage, given the year following the teaching of the material, some changes of presentation and motivation need be made. Of the 10,091 errors and omissions, 2617 or 25.9% were attributed to the misplacing of the decimal point. If we count only the errors due to the

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15. George Hastings McNair. Methods of Teaching Modern Day Arithmetic. (Boston, Richard Badger, 1923), p. 183.

incorrect placing of the decimal point and eliminate all other errors, we would have a score of only 85.0%. Let us try to build, within the experience of the child, an understanding of locating the decimal point. It seems that in the past, we have been relying too frequently upon rote learning and, if this fails, the pupil is lost in the procedure, which should follow. A thorough knowledge of this subject will be possible by revising our teaching methods and through a greater concentration of time on the fundamentals of percentage.

APPENDIX

Fundamentals of Percentage

Name _____ Date _____ Boy or girl? _____

When is your next birthday? _____ How old will you be at that time? _____

Type of school where you attended the seventh grade. Graded _____ Rural _____ Junior High _____

Grade you received in your seventh grade mathematics _____

Instructions: In working the following problems solve the easier problems first and then refer to the remaining problems. All work is to be shown on these sheets. You are not to use any additional paper. Read the problem carefully before you begin to work. Write your answers on the lines provided for the answering. You are to have forty minutes which will give plenty of time.

PART I

Changing fractions to percent—Example: ½ = 50%

1. ¼ = _____%
2. ⅝ = _____%
3. 4½ = _____%
4. 12 = _____%

Changing percents to fractions or mixed numbers—Example: 15% = 3/20

5. 1% = _____
6. 37½% = _____
7. 325% = _____
8. 12½% = _____

Changing decimals to percents—Example: .27 = 27%

9. .35 = _____%
10. 3.06 = _____%
11. .0026 = _____%
12. 30 = _____%

Changing percents to decimals—Example: 26% = .26

13. 4.5% = _____
14. 32% = _____
15. .003% = _____
16. 521.6% = _____

PART II

Instructions: Solve the following problems. Show all work for the problem in the space following the problem and put the answer in the space provided

17. 14% of 76 = _____
18. 7% of _____ = 42
19. 2 = _____% of 100
20. 420 = 105% of _____
21. _____% of 85 = 8.5
22. 119.5% of 2000 = _____
23. .03% of .16 = _____
24. .325% of _____ = 1.5
25. .12% of _____ = .016

26. How much is saved by purchasing an article listed at $5.20 at 25% off?

BIBLIOGRAPHY

Barber, Harry C. Junior High School Mathematics. Cambridge, Mass., Riverside Press. 1927. Seventh School Year, 226 pages.

Examined for material dealing with percentage.

Brueckner, Leo J. The New Curriculum Arithmetic. Chicago, John Winston Co. 1939. Grade 7.

State adopted text.

Brueckner and Melby. Diagnostic and Remedial Teaching. Chicago, Houghton and Mifflin Co. 1931. 598 p.

Diagnosis in teaching.

Dickey, John. "The Value of Estimating Answers to Arithmetic Problems and Examples". (Elementary School Journal, XXXV, Sept. 1934).

Value of estimating answers.

Edwards, Algae. A Comparison of Junior High School and Elementary Seventh Grade Pupils in Achievement and Growth in Arithmetic. Masters 1937. Sam Houston St. T. C. 69 p ms.

A research study in arithmetic.

Good, Barr, and Scates. The Methodology of Research. New York, D. Appleton-Century Co. 1935. 882 p.

Procedure in research.

Justice, James Carlos. An Analysis of One Hundred Eighty-eight Seventh, Eighth, and Ninth Grade Pupils in Arithmetic, Reading and English. Masters 1940, Wittenberg. 90 p ms.

Study in pupil preparation.

McNair, George Hastings. Methods of Teaching Modern Day Arithmetic. Boston, Richard Badger 1923. 419 p.

Methods of teaching arithmetic.

Mallory, Cooke and Loughren, Using Arithmetic Series. Chicago, Benj. H. Sanborn and Co. 1941. 348 p.

State adopted text.

Reeve, W. D. "Attacks on Mathematics and How to Meet Them". National Council for Teachers of Mathematics. 11th yearbook.

Common difficulties found in arithmetic.

Ruch, Knight, and Studebaker. "Basic Facts of Percentage". Compass Diagnostic Test.

This test was reviewed for content of material.

Tests, Scales, Books, and Measuring Devices for Schools and Industry. Cincinnati, C. A. Gregory, Cat. #42.

Source of standardized tests.

Thomson, Jeannie B. The Art of Teaching Arithmetic. New York, Longman, Green, and Co. 1926.

Arithmetic teaching.

Thorndike, Edward L. The New Methods in Arithmetic. New York, Rand McNally and Co. 1911.

Methods of teaching arithmetic.