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A Study In Retention In Fundamental Operations In Algebra

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A STUDY IN RETENTION IN FUNDAMENTAL OPERATIONS IN ALGEBRA

being

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by

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CHAPTER I

INTRODUCTION

In past years a number of surveys have been made on the subject of retention. The surveys have covered practically all of the subjects in our school curriculum. Some surveys have been made along this line in the field of mathematics. However, most of these cover the higher branches of mathematics. Even those covering algebra delve into the more complex forms of the subject. Therefore it was decided that a survey in retention based on the elementary phase would be worthwhile.

Retention in learning is the final measuring stick for finding how well subject matter is taught. Practically all subjects in the high school and grade school curriculum, as well as that of the institutions of higher learning, are dependent on some subject which was studied by the student during the preceding years. This makes it necessary that one of two conditions must be met (1) Either the student must retain sufficient knowledge of the preceding work to carry on the advanced work satisfactorily, or (2) A review of the preceding work must be given.

Problem

The problem is to discover as nearly as possible, "The retention of sophomores, who the preceding year had taken algebra, in the four fundamental arithmetic operations of positive and negative numbers."

The four fundamental arithmetic operations are namely: addition, subtraction, multiplication and division. In making computations with positive and negative numbers there are four possibilities involving cases concerning a positive and a negative, a negative and a positive, two negative, and two positive numbers. Negative numbers are numbers having a minus value and positive numbers are numbers which have a plus value.

Purpose

The purpose of the test was fourfold. First, to ascertain the knowledge retained by students in each of three classes of high schools in Kansas. The classes chosen were A, B and C and are accredited as such by the state department of education. Some of the factors used in accrediting schools in Kansas are: enrollment, school plant, library, number and qualifications of teachers and teacher tenure. The classification of the various schools participating was taken from the Kansas Educational Direct-

ory for 1943-1944.[1]

Second, to determine the relation of the amount retained to the natural intelligence of the pupil. The natural intelligence of the pupil was procured from each school and was designated by a score received on a standard I. Q. test.

Third, to compare the amount of knowledge retained with the pupils scholastic grade received in the subject.

Fourth, to ascertain the amount retained by boys and the amount retained by girls in each of the school classifications.

Method and Scope

A test was formulated which would eliminate as many mechanical errors as possible. Only the simple numbers sixteen and four and eighteen and three were used. The numbers sixteen and four were arranged horizontally in four rows, each row consisting of one each of the four possibilities involved. The numbers eighteen and three were arranged vertically in the same manner. By this method each operation was performed twice.

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1. George L. McClenny, Kansas Educational Directory, Kansas State Department of Education, Topeka, State Printer, 1943, pp. 20-71.

After the tests were prepared and printed a postal card questionnaire was sent to seventy-one schools asking if they would cooperate in administering the test. These seventy-one schools were in each of the three classifications as shown in the Kansas School Directory for 1943-44. It was explained that the test was simple and could be given in five or ten minutes. The questionnaire was sent on a double postal card. All that the administrator need do was mark the card either yes or no and return it. Of the seventy-one cards sent sixty were returned. Nine answered no without explanation and seven answered no and explained. Four of these said that algebra was alternated with another subject and it had not been taught the year before. Three said that the algebra teacher had been so poor the year before that it was inadvisable to use their school in the testing. The remaining forty-four schools requested a total of seven hundred and thirty tests. These were sent and forty-three of these schools returned them. Postage was included in each package of tests for their return. Six hundred and two tests were returned. The reason for the difference in number sent and number returned is accounted for in the fact that one school did not return the tests and several of the others did not use as many as were requested.

Review of Literature

A thorough search was made of available literature pertaining to the study of retention in algebra. There was considerable information concerning the more advanced phases in the field, but very little work was found dealing with such an elementary problem as positive and negative numbers.

The literature available, which was of a comparable nature to the problem at hand, was in regard to the second purpose of this thesis or a study of retention as compared with I. Q.'s.

White says, "What will it profit a pupil, even if he has a great mind, if he is not interested in his work, does not put forth the proper effort and has no purpose in view."[2] This statement was made in relation to proof that a high I. Q. was not a definite index to a person's ability to master elementary algebra.

Mason takes the same point of view in the statement, "Students with a high I. Q., who were tested at the completion of the ninth grade course in mathematics, tend to

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2. Annabel L. White, The Retention of Elementary Algebra . . . After Varying Intervals of Time. (John Hopkins University, 1930), p. 47.

forget more, but they can afford to forget more and still rank high."[3]

Layton takes a somewhat different point of view in the statement, "There is some evidence that the ranking of pupils according to I. Q.'s and ranking according to knowledge retained tends to be similar."[4]

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3. E. T. Layton, "Persistence of Learning in Elementary Algebra." (Journal of Educational Psychology Vol. 23, p. 51, January 1932).

4. Nellie C. Mason, A Study in the Retention of Junior High School Mathematics. Master's Thesis, University of Minnesota, 1932, p. 26.

CHAPTER II

COMPARISONS OF RETENTION

Retention by all Students

Before separating the materials collected into the various classifications that were to be studied and making an analysis of the same, an overall view of the picture was studied. The test which was sent out to the students was composed of thirty-two problems. There were eight problems in each of the sections, addition, subtraction, multiplication, and division. This method provided for the working of each problem twice, once vertically and once horizontally, using different although elementary numbers in each case. This was done for the purpose of eliminating mechanical errors. If the student performed the operation correctly in one case and incorrectly in the other, the mistake was probably a mechanical error. If the problem was solved correctly in both cases the indication was that there was an understanding of the principle involved. The six hundred and two tests with thirty-two problems in a test made a total of nineteen thousand two hundred and sixty-four problems. Of the total number of problems fifteen thousand and twenty were correctly solved. Figuring this on a percentage basis it revealed

that the percent that were correctly solved was seventy-seven and eight-tenths.

Breaking the test down into its parts shows that there were four thousand nine hundred and four addition problems of which three thousand seven hundred and sixty-two were answered correctly. This means that seventy-six and seven-tenths percent of the addition problems were correct.

Three thousand one hundred and forty-four of the subtraction problems were correct or sixty-four and one-tenth percent.

Four thousand one hundred and twelve of the multiplication problems were correct or eighty-three and nine-tenths percent.

Four thousand three hundred and fifty-f ur division problems were correct or eighty-eight and eight-tenths percent.

If the test is reduced further it shows that there were one thousand two hundred and twenty-six addition problems involving the adding of a positive to a negative. Nine hundred and twenty-eight of these were answered correctly or seventy-five and six-tenths percent.

Eight hundred and seventy-two problems involving adding a negative to a positive were answered correctly

or seventy-one and one-tenth percent.

One thousand one hundred and forty-six problems involving the addition of two positive numbers were correctly answered or ninety-three and four-tenths percent.

Eight hundred and sixteen problems involving the addition of two negative numbers were correct or sixty-six and five-tenths percent.

Six hundred and forty-two involving the subtraction of a positive from a negative were correct or fifty-two and three-tenths percent.

Seven hundred and seven problems involving the subtracting a negative from a positive were correct or fifty-seven and six-tenths percent.

Nine hundred and fifty-five problems involving subtracting a positive from a positive were correct or seventy-seven and nine-tenths percent.

Eight hundred and forty problems involving subtracting a negative from a negative were correct or sixty-eight and five-tenths percent.

The multiplication problems disclosed that one thousand and seventy-seven or eighty-seven and eight-tenths percent correctly solved problems involving multiplication of a negative by a positive.

Most of the multiplication problems missed involved multiplying a positive by a negative. Only nine hundred

and six problems or seventy-three and nine-tenths percent were answered correctly.

The best multiplication result was found where it would naturally be expected, in problems dealing with the multiplication of a positive by a positive. There were one thousand one hundred and two correct answers or eighty-nine and nine-tenths percent.

One thousand and twenty seven problems involving multiplying a negative by a negative or eighty-three and seven-tenths percent were answered correctly.

The best results of the test were discovered in the division problems. In problems involving dividing a negative by a positive one thousand one hundred and fifty-nine or ninety-four and five-tenths percent were correct.

Only nine hundred and thirty-four could correctly solve a positive divided by a negative. This was seventy-six and one-tenth percent.

The most accurate work of the test was done in dividing a positive. One thousand one hundred and seventy-five solved this section correctly or a percentage of ninety-five and eight-tenths.

One thousand and eighty-six correctly solved problems involving the division of a negative by a negative or eighty-eight and five-tenths percent.

TABLE I

PERCENTAGE OF CORRECT ANSWERS IN EACH POSSIBLE CASE

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	75.6%	71.1%	93.4%	66.5%
Subtraction	52.3%	57.6%	77.9%	68.5%
Multiplication	87.8%	73.9%	89.9%	83.7%
Division	94.5%	76.1%	95.8%	88.5%

Retention as Shown In The Three School Classifications

The three school classifications in Kansas as accredited by the State Department of Education are A, B, and C. The method of accrediting is explained on page 2.

The actual results of the test are as follows:

Two hundred and eighty-two students from Class A accredited schools took the test, which made nine thousand and twenty-four problems. There were two thousand two hundred and fifty-six problems in each of the four arithmetic processes.

One thousand seven hundred and nine addition problems were worked correctly, or seventy-six percent.

One thousand four hundred and seven subtraction problems were correct, or sixty-two percent.

One thousand eight hundred and eighty-two multipli-

cation problems were correct or eighty-three percent.

Two thousand and twenty-three division problems were correct or eighty-nine percent.

Among the addition problems there were five hundred and sixty-four each involving adding a positive to negative, a negative to a positive, a positive to a positive and a negative to a negative.

Four hundred and sixteen involving adding a positive to a negative were correct or seventy-three and seven-tenths percent.

Three hundred and eighty-seven involving adding a negative to a positive were correct or sixty-eight and six-tenths percent.

Five hundred and thirty-four involving adding a positive to a positive were correct or ninety-four and seven-tenths percent.

Three hundred and seventy-two involving adding a negative to a negative were correct or sixty-five and nine-tenths percent.

In the subtraction problems two hundred and eighty-three involving subtracting a positive from a negative were correct or fifty and two-tenths percent.

Three hundred and twelve involving subtracting a negative from a positive or fifty-five and three-tenths

percent.

Four hundred and thirty involving subtracting a positive from a positive were correct or seventy-six and six-tenths percent.

Three hundred and eighty-two involving subtracting a negative from a negative were correct or sixty-seven and seven-tenths percent.

Five hundred and one involving multiplying negative by a positive were correct or eighty-eight and eight-tenths percent.

Four hundred and nine involving multiplying a positive by a negative were correct or seventy-two and five-tenths percent.

Five hundred and seven involving multiplying a positive by a positive were correct or eighty-nine and nine-tenths percent.

Four hundred and sixty-five involving multiplying a negative by a negative were correct or eighty-two and four-tenths percent.

Five hundred and thirty-six involving dividing a positive by a negative were correct or ninety-five percent.

Four hundred and thirty-nine involving dividing a negative by a positive were correct or seventy-seven and eight-tenths percent.

Five hundred and forty-four involving dividing a positive by a positive were correct or ninety-six and four-tenths percent.

Five hundred and four involving dividing a negative by a negative were correct or eighty-nine and three-tenths percent.

TABLE II

PERCENTAGE OF CORRECT ANSWERS IN EACH OF THE POSSIBLE CASES IN CLASS A HIGH SCHOOLS

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	73.7%	68.6%	94.7%	65.9%
Subtraction	50.2%	55.3%	76.6%	67.7%
Multiplication	88.8%	72.5%	89.9%	82.4%
Division	95%	77.8%	96.4%	89.3%

In schools, which were accredited as class B, one hundred and eighty-five students took the test, which totaled five thousand nine hundred and twenty problems. These were, like those in class A, divided into addition, subtraction, multiplication and division.

Of the one thousand four hundred and eighty addition problems one thousand one hundred and thirty-eight were solved correctly or seventy-six and nine-tenths percent.

Nine hundred and sixty subtraction problems were solved correctly or sixty-four and eight-tenths percent.

One thousand two hundred and thirty-seven multiplication problems were solved correctly or eighty-three and five-tenths percent.

One thousand three hundred and one division problems were solved correctly or eighty-seven and nine-tenths percent.

The four sets were further divided, as they were in the class A schools, into the four possibilities in each process.

There were three hundred and seventy problems involving adding a positive to a negative. There were two hundred and eighty-six answered correctly or seventy-seven and two-tenths percent.

Two hundred and sixty-three problems involving adding a negative to a positive were answered correctly or seventy-one and nine-tenths percent.

Three hundred and forty-four problems involving adding a positive to a positive were answered correctly or ninety-two and nine-tenths percent.

Two hundred and forty-five problems involving adding a negative to a negative were correct or sixty-six and two-tenths percent.

In subtraction one hundred and ninety-six problems involving subtracting a positive from a negative were correct or fifty-two and nine-tenths percent.

Two hundred and twenty-two problems involving subtracting a negative from a positive were correct or sixty percent.

Two hundred and ninety problems involving subtracting a positive from a positive were correct or seventy-eight and three-tenths percent.

Two hundred and fifty-two problems involving subtracting a negative from a negative were correct or sixty-eight and one-tenth percent.

Three hundred and nineteen problems involving multiplying a negative by a positive were correct or eighty-six and two-tenths percent.

Two hundred and seventy-five problems involving multiplying a positive by a negative were correct or seventy-eight and three-tenths percent.

Three hundred and twenty-nine problems involving multiplying a positive by a positive were correct or eighty-eight and nine-tenths percent.

Three hundred and fourteen problems involving multiplying a negative by a negative were correct or eighty-four and eight-tenths percent.

Three hundred and forty-six problems involving dividing a negative by a positive were correct or ninety-three and five-tenths percent.

Two hundred and seventy-nine problems involving dividing a negative by a positive were correct or seventy-five and four-tenths percent.

Three hundred and forty-nine problems involving dividing a positive by a positive were correct or ninety-four and three-tenths percent.

Three hundred and twenty-seven problems involving dividing a negative by a negative were correct or eighty-eight and three-tenths percent.

TABLE III

PERCENTAGE OF CORRECT ANSWERS IN EACH OF THE POSSIBLE CASES IN CLASS B HIGH SCHOOLS

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	77.2%	71.9%	92.9%	66.2%
Subtraction	52.9%	60%	78.3%	68.1%
Multiplication	86.2%	78.3%	88.9%	84.8%
Division	93.5%	75.4%	94.3%	88.3%

In schools that were accredited as class C, one hundred and thirty-five students took the test, which totaled four thousand three hundred and twenty problems.

These were, like both of the others, divided into addition, subtraction, multiplication and division.

Of the one thousand and eighty addition problems eight hundred and twenty-seven were solved correctly or seventy-six and five-tenths percent.

Six hundred and ninety subtraction problems were correct or fifty-four and six-tenths percent.

Nine hundred and five multiplication problems were correct or eighty-three and seven-tenths percent.

Nine hundred and forty-two division problems were correct or eighty-seven and two-tenths percent.

There were two hundred and seventy problems involving addition of a positive to a negative. There were two hundred and four of these correct or seventy-five and five-tenths percent.

There were two hundred correct problems involving addition of a negative to a positive or seventy-four percent.

There were two hundred and forty-six correct problems involving adding a positive to a positive or ninety-one and one-tenth percent.

There were one hundred and seventy-seven correct problems involving adding a negative to a negative or sixty-five and five-tenths percent.

There were one hundred and forty-one correct problems involving subtracting a positive from a negative or forty-two and two-tenths percent.

There were one hundred and fifty-two correct problems involving subtracting a negative from a positive or fifty-six and two-tenths percent.

There were two hundred and thirteen correct problems involving subtracting a positive from a positive or seventy-eight and eight-tenths percent.

There were one hundred and eighty-four correct problems involving subtracting a negative from a negative or sixty-eight and one-tenth percent.

There were two hundred and thirty-five correct problems involving multiplying a negative by a positive or eighty-seven percent.

There were two hundred correct problems involving multiplying a positive by a negative or seventy-four percent.

There were two hundred and forty-four correct problems involving multiplying a positive by a positive or ninety and three-tenths percent.

There were two hundred and twenty-six correct problems involving multiplying a negative by a negative or eighty-three percent.

There were two hundred and fifty-five correct problems involving dividing a positive to a negative or ninety-four and five-tenths percent.

There were one hundred and ninety-four correct problems involving dividing a negative by a positive or seventy-one and eight-tenths percent.

There were two hundred and sixty correct problems involving dividing a positive by a positive or ninety-six and two-tenths percent.

There were two hundred and thirty-three correct problems involving dividing a negative by a negative or eighty-six and two-tenths percent.

TABLE IV

PERCENTAGE OF CORRECT ANSWERS IN EACH OF THE POSSIBLE CASES IN CLASS C HIGH SCHOOLS

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	75.5%	74%	91.1%	65.6%
Subtraction	52.2%	56.2%	78.8%	68.1%
Multiplication	87%	74%	90.3%	83%
Division	94.5%	71.8%	96.2%	86.2%

Retention As Revealed With Students I. Q.

On the test sheet was a space provided for the student's

I. Q. A great many papers were returned with this space filled in with average, low, high, etc. However, two hundred and fifty-three were returned with figures which might be used. These papers were divided into three groups. Papers of students whose I. Q. was ninety or below were placed in one group, those between ninety and one hundred and ten in another and those over one hundred ten in a third.

In the group with an I. Q. of over one hundred and ten there were seventy-six students. Two thousand and seventy-three answers were correct out of a possible two thousand four hundred and thirty-two or eighty-five and two-tenths percent.

In the group with an I. Q. of between ninety and one hundred and ten there were one hundred an forty-five students who correctly answered three thousand five hundred and thirty-two of the four thousand six hundred and forty problems or seventy-six percent.

In the group with an I. Q. of ninety or less there were thirty-two students who correctly answered six hundred and fifty-six of the one thousand and twenty-four problems or sixty-three and six-tenths percent.

These three groups were further divided into groups containing problems involving only one of the four funda-

mental arithmetic operations.

In the lower I. Q. group the thirty-two students correctly answered one hundred and forty-eight of the two hundred and fifty-six problems involving addition or fifty-seven and eight-tenths percent.

This section was further divided into groups containing problems involving adding a positive to a negative, negative to positive, positive to positive and negative to negative. There were sixty-four problems in each group.

Thirty-six of the problems involving adding a positive to a negative were correct or fifty-six and two-tenths percent.

Thirty of the problems involving adding a negative to a positive were correct or forty-seven percent.

Fifty-one of the problems involving adding a positive to a positive were correct or seventy-nine and seven-tenths percent.

Thirty-one of the problems involving adding a negative to a negative were correct or forty-seven percent.

One hundred and twenty-one of the two hundred and fifty-six problems involving subtraction were correct or forty-seven and two-tenths percent.

Twenty-three of the problems involving subtraction of a positive from a negative were correct or thirty-six

percent.

Twenty-five of the problems involving subtraction of a negative from a positive were correct or thirty-nine and five-tenths percent.

Thirty-eight of the problems involving subtraction of a positive from a positive were correct or fifty-nine and three-tenths percent.

Thirty-five of the problems involving subtraction of a negative from a negative were correct or fifty-four and seven-tenths percent.

One hundred and seventy-one of the two hundred and fifty-six multiplication problems were correct or sixty-six and eight-tenths percent.

Forty-two of the problems involving multiplying a negative by a positive were correct or sixty- ive and six-tenths percent.

Thirty-eight of the problems involving multiplying a positive by a negative were correct or fifty-nine and three-tenths percent.

Forty-five of the problems involving multiplying a positive by a positive were correct or seventy and three-tenths percent.

Forty-six of the problems involving multiplying a negative by a negative were correct or seventy-one and

eight-tenths percent.

Two hundred and thirteen of the two hundred and fifty-six division problems were correct or eighty-three and two-tenths percent.

Fifty-four of the problems involving division of a negative by a positive were correct or eighty-four and three-tenths percent.

Forty-five of the problems involving division of a positive by a negative were correct or seventy and three-tenths percent.

Fifty-five of the problems involving division of a positive by a positive were correct or eighty-five and nine-tenths percent.

Fifty-nine of the problems involving division of a negative by a negative were correct or ninet -two and one-tenth percent.

TABLE V

PERCENTAGE OF CORRECT ANSWERS, IN EACH OF THE FOUR CASES, OF STUDENTS WITH AN I. Q. BELOW NINETY

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	56.2%	47%	79.7%	47%
Subtraction	36%	39.5%	59.3%	54.7%
Multiplication	65.6%	59.3%	70.3%	71.8%
Division	84.3%	70.3%	85.9%	92.1%

There were one hundred and forty-five students in the middle group whose I. Q.'s were from ninety to one hundred and ten. These students correctly worked six hundred and fifty-five of the one thousand and sixty addition problems.

One hundred and eighty-four problems involving adding a positive to a negative were correct or sixty-nine and four-tenths percent.

One hundred and sixty-seven problems involving adding a negative to a positive were correct or sixty-three percent.

Two hundred and forty-three problems involving adding a positive to a positive were correct or ninety and five-tenths percent.

One hundred and sixty-one problems involving adding a negative to a negative were correct or sixty and eight-tenths percent.

Five hundred and ninety-eight subtraction problems were correct or fifty-six and four-tenths percent.

One hundred and ten problems involving subtracting a positive from a negative were correct or forty-one and five-tenths percent.

One hundred and thirty-six problems involving subtracting a negative from a positive were correct or fifty-

one and three-tenths percent.

One hundred and ninety-five problems involving subtracting a positive from a positive were correct or seventy-three and six-tenths percent.

One hundred and fifty-seven problems involving subtracting a negative from a negative were correct or sixty percent.

Eight hundred and sixty of the multiplication problems were correct or eighty-one and one-tenth percent.

Two hundred and thirty-two problems involving multiplying a negative by a positive were correct or eighty-seven and five-tenths percent.

One hundred and seventy-nine problems involving multiplying a positive by a negative were correct or sixty-seven and five-tenths percent.

Two hundred and thirty-four problems involving multiplying a positive by a positive were correct or eighty-eight and three-tenths percent.

Two hundred and fifteen problems involving multiplying a negative by a negative were correct or eighty-one and one tenth percent.

Nine hundred and nineteen division problems were correct or eighty-six and seven-tenths percent.

Two hundred and forty-eight problems involving

division of a negative by a positive were correct or ninety-three and five-tenths percent.

One hundred and ninety-five problems involving division of a positive by a negative were correct or seventy-three and five-tenths percent.

Two hundred and fifty problems involving division of a positive by a positive were correct or ninety-four and three-tenths percent.

Two hundred and twenty-six problems involving division of a negative by a negative were correct or eighty-five and three-tenths percent.

TABLE VI

PERCENTAGE OF CORRECT ANSWERS, IN EACH OF THE FOUR CASES, OF STUDENTS WITH AN I.Q. BETWEEN 90-110

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	69.4%	63%	90.5%	60.8%
Subtraction	41.5%	51.3%	73.6%	60%
Multiplication	87.5%	67.5%	88.3%	81.1%
Division	93.5%	73.5%	94.3%	85.3%

There were seventy-six students in the group whose I. Q. was more than one hundred and ten.

This group worked five hundred and ten of the six hundred and eight addition problems correctly or eighty-

three and nine-tenths percent.

One hundred and twenty-eight problems involving addition of a positive to a negative were correct or eighty-three and nine-tenths percent.

One hundred and twenty-one problems involving addition of a negative to a positive were correct or seventy-nine and six-tenths percent.

One hundred and forty-five problems involving addition of a positive to a positive were correct or ninety-five and four-tenths percent.

One hundred and sixteen problems involving addition of a negative to a negative were correct or seventy-six and three-tenths percent.

Four hundred and forty-three subtraction problems were correct or seventy-two and eight-tenths percent.

Ninety-two problems involving subtraction of a positive from a negative were correct or sixty and five-tenths percent.

One hundred and six problems involving subtraction of a negative from a positive were correct or sixty-nine and six-tenths percent.

One hundred and twenty-six problems involving subtraction of a positive from a positive were correct or eighty-two and nine-tenths percent.

One hundred and nineteen problems involving subtraction of a negative from a negative were correct or seventy-eight and three-tenths percent.

Five hundred and forty-six multiplication problems were correct or eighty-nine and eight-tenths percent.

One hundred and thirty-seven problems involving multiplication of a negative by a positive were correct or ninety and one-tenth percent.

One hundred and thirty-five problems involving multiplication of a positive by a negative were correct or eighty-eight and eight-tenths percent.

One hundred and thirty-nine problems involving multiplication of a positive by a positive were correct or ninety-one and four-tenths percent.

One hundred and fifty-two problems i volving multiplication of a negative by a negative were correct or eighty-eight and eight-tenths percent.

Five hundred and seventy-four division problems were correct or ninety-four and four-tenths percent.

One hundred and forty-eight problems involving division of a negative by a positive were correct or ninety-seven and three tenths percent.

One hundred and thirty-five problems involving division of a positive by a negative were correct or eighty-

eight and eight-tenths percent.

One hundred and forty-nine problems involving division of a positive by a positive were correct or ninety-eight percent.

One hundred and forty-two problems involving division of a negative by a negative were correct or ninety-three and four-tenths percent.

TABLE VII

PERCENTAGE OF CORRECT ANSWERS, IN EACH OF THE FOUR CASES, OF STUDENTS WITH AN I.Q. OVER 110

	Negative and Positive	Positive and Negative	Positive and Positive	Negative and Negative
Addition	83.9%	79.6%	95.4%	76.3%
Subtraction	60.5%	69.6%	82.9%	78.3%
Multiplication	90.1%	88.8%	91.4%	88.8%
Division	97.3%	88.8%	98%	93.4%

Retention Compared With Scholastic Grade

The six hundred and two, who were tested, received the following grades: Eighty-two, A; one hundred and forty-seven, B; two hundred and fifteen, C; one hundred and twenty-four, D; and thirty-four, F.

Students, who had received a letter grade of A, correctly worked six hundred and fifteen of the six hundred

and fifty-six addition problems correctly or ninety-three and seven-tenths percent.

Five hundred and sixty-three of the subtraction problems were correct or eighty-five and eight-tenths percent.

Six hundred and twenty-seven of the multiplication problems were correct or ninety-five and six-tenths percent.

Six hundred and forty-four of the division problems were correct or ninety-nine and seven-tenths percent.

Students, who had received a letter grade of B, correctly worked one thousand and forty-four of the one thousand one hundred and seventy-six addition problems or eighty-eight and eight-tenths percent.

Eight hundred and ninety-one of the subtraction problems were correct or seventy-five and eight-tenths percent.

One thousand and sixty of the multiplication problems were correct or ninety and two-tenths percent.

One thousand and eighty-six of the division problems were correct or ninety-four and four-tenths percent.

Students, who had received a letter grade of C, correctly worked one thousand two hundred and sixty-seven of the one thousand seven hundred and twenty addition

problems or seventy-three and seven-tenths percent.

One thousand and fifty-two of the subtraction problems were correct or sixty-one and two-tenths percent.

One thousand four hundred and sixty-seven of the multiplication problems were correct or eighty-five and three-tenths percent.

One thousand five hundred and forty-two of the division problems were correct or eighty-nine and six-tenths percent.

Students who had received a letter grade of D, correctly worked five hundred and eighty of the nine hundred and ninety-two addition problems or fifty-eight and five-tenths percent.

Four hundred and twenty of the subtraction problems were correct or forty-two and three-tenths p rcent.

Six hundred and eighty of the multiplication problems were correct or sixty-eight and four-tenths percent.

Seven hundred and eighty-one of the division problems were correct or seventy-eight and seven-tenths percent.

Students, who had received a failing grade, correctly worked seventy-one of the one hundred and ninety-two addition problems or thirty-six and nine-tenths percent.

Forty-four of the subtraction problems were correct or twenty-two and nine-tenths percent.

One hundred and three of the multiplication problems were correct or fifty-three and six-tenths percent.

One hundred and twenty-nine of the division problems were correct or sixty-seven and three-tenths percent.

TABLE VIII

PERCENTAGE OF THE PROBLEMS CORRECTLY SOLVED BY THOSE RECEIVING EACH OF THE FIVE LETTER GRADES

	Addition	Subtraction	Multiplication	Division
A	93.7%	85.8%	95.6%	99.7%
B	88.8%	75.8%	90.2%	94.4%
C	73.7%	61.2%	85.3%	89.6%
D	58.5%	42.3%	68.4%	78.7%
F	36.9%	22.9%	53.6%	67.3%

In class A schools students, who had received a letter grade of A, correctly worked three hundred and twenty-nine of the three hundred and forty-four addition problems or ninety-five and six-tenths percent.

Three hundred and twelve of the subtraction problems were correct or ninety and seven-tenths percent.

Three hundred and twenty-nine of the multiplication problems were correct or ninety-five and six-tenths percent.

Three hundred and forty-one of the division problems were correct or ninety-nine and one-tenth percent.

Students, who had received a letter grade of B, correctly worked five hundred and one of the five hundred and fifty-two addition problems or ninety and seven-tenths percent.

Four hundred and twenty-four of the subtraction problems were correct or seventy-eight and six-tenths percent.

Five hundred and four of the multiplication problems were correct or ninety and eight-tenths percent.

Five hundred and twenty-four of the division problems were correct or ninety-four and four-tenths percent.

Students, who received a letter grade of C, correctly worked five hundred and sixty-five of the eight hundred and sixteen addition problems or sixty-eight percent.

Four hundred and fifty-five of the subtraction problems were correct or fifty-five and eight-tenths percent.

Six hundred and seventy-one of the multiplication problems were correct or eighty-two and two-tenths percent.

Seven hundred and thirty-six of the division problems were correct or ninety and one-tenth percent.

Students, who had received a letter grade of D, correctly worked two hundred and seventy-four of the four hundred and sixty-four addition problems or fifty-nine percent.

One hundred and eighty-five of the subtraction problems were correct or thirty-nine and eight-tenths percent.

Three hundred and fifteen of the multiplication problems were correct or sixty-seven and nine-tenths percent.

Three hundred and sixty-one of the division problems were correct or seventy-seven and eight-tenths percent.

Students, who received a failing grade, correctly worked forty of the eighty addition problems or fifty percent.

Twenty-eight of the subtraction problems were correct or thirty-five percent.

Fifty-five of the multiplication problems were correct or sixty-eight and seven-tenths percent.

Sixty-three of the division problems were correct or seventy-eight and seven-tenths percent.

TABLE IX

PERCENTAGE OF CORRECT ANSWERS,

BY LETTER GRADE, IN CLASS A SCHOOLS

	Addition	Subtraction	Multiplication	Division
A	95.6%	90.7%	95.6%	99.1%
B	90.7%	78.6%	90.8%	94.4%
C	68%	55.8%	82.2%	90.1%
D	59%	39.8%	67.9%	77.8%
F	50%	35%	68.7%	78.7%

In the class B schools students, who received a letter grade of A, correctly worked one hundred and seventy-three of the one hundred and ninety-two addition problems or ninety and one-tenth percent.

One hundred and forty-five of the subtraction problems were correct or seventy-five and one-ten h percent.

One hundred and eighty-eight of the multiplication problems were correct or ninety-two and seven-tenths percent.

One hundred and eighty-three of the division problems were correct or ninety-five and three-tenths percent.

Students, who received a letter grade of B, correctly worked three hundred and sixty-five of the four hundred and eight addition problems or eighty-nine and four-tenths percent.

Three hundred and twenty-five of the subtraction problems were correct or seventy-nine and six-tenths percent.

Three hundred and fifty-five of the multiplication problems were correct or eighty-seven and four-tenths percent.

Three hundred and sixty-four of the division problems were correct or eighty-nine and two-tenths percent.

Students, who had received a letter grade of C, correctly worked three hundred and sixty of the four hundred and seventy-two addition problems or seventy-six and two-tenths percent.

Three hundred and nineteen of the subtraction problems were correct or sixty-seven and five-tenths percent.

Four hundred and twenty-four of the multiplication problems were correct or eighty-nine and eight-tenths percent.

Four hundred and twenty-eight of the division problems were correct or ninety and six-tenths percent.

Students, who received a grade of C, correctly worked one hundred and sixty-nine of the three hundred and four addition problems or fifty-five and five-tenths percent.

One hundred and twenty-eight of the subtraction

problems were correct or forty-two and one-tenth percent.

Two hundred and thirty-five of the division problems were correct or eighty-three and four-tenths percent.

Students, who received a failing grade, correctly worked fifty-three of the one hundred and four addition problems or fifty and nine-tenths percent.

Forty-four of the subtraction problems were correct or forty-two and three-tenths percent.

Two hundred and thirty-five of the division problems were correct or eighty-three and four-tenths percent.

Fifty-six of the multiplication problems were correct or fifty-three and eight-tenths percent.

Sixty-nine of the division problems were correct or sixty-six and three-tenths percent.

TABLE X

PERCENTAGE OF CORRECT ANSWERS,

BY LETTER GRADE IN CLASS B SCHOOLS

	Addition	Subtraction	Multiplication	Division
A	90.1%	75.1%	92.7%	95.3%
B	89.4%	79.6%	87.4%	89.2%
C	76.2%	67.5%	89.8%	90.6%
D	55.5%	42.1%	77.3%	83.4%
F	50.9%	42.3%	53.8%	66.3%

In class C schools students, who had received a letter grade of A, correctly worked one hundred and

thirteen of the one hundred and twenty addition problems or ninety-four and two-tenths percent.

One hundred and six of the subtraction problems were correct or eighty-eight and three-tenths percent.

This group of students worked all of the problems in multiplication and division correctly.

Students, who had received a letter grade of B, correctly worked one hundred and seventy-nine of the two hundred and sixteen addition problems or eighty-two and four-tenths percent.

One hundred and forty-two of the subtraction problems were correct or sixty-five and four-tenths percent.

Two hundred and one of the multiplication problems were correct or ninety-three percent.

One hundred and ninety-eight of the di ision problems were correct or ninety-one and seven-tenths percent.

Students, who had received a letter grade of C, correctly worked three hundred and forty-four of the four hundred and thirty-two addition problems or seventy-nine and six-tenths percent.

Two hundred and seventy-eight of the subtraction problems were correct or sixty-four and three-tenths percent.

Three hundred and seventy-two of the multiplication

problems were correct or eighty-six and one-tenth percent.

Three hundred and seventy-eight of the division problems were correct or eighty-seven and five-tenths percent.

Students, who received a letter grade of D, correctly worked one hundred and thirty-seven of the two hundred and twenty-four addition problems or sixty-one and one-tenth percent.

One hundred and seven of the subtraction problems were correct or forty-seven and seven-tenths percent.

One hundred and forty of the multiplication problems were correct or sixty-two and five-tenths percent.

One hundred and sixty-seven of the division problems were correct or seventy-four and five-tenths percent.

Students, who received a failing grade, correctly answered fifty-eight of the eighty-eight addition problems or sixty-five and nine-tenths percent.

Fifty-two of the subtraction problems were correct or fifty-nine and one-tenth percent.

Seventy-two of the multiplication problems were correct or eighty-one and seven-tenths percent.

TABLE XI

PERCENTAGE OF CORRECT ANSWERS,

BY LETTER GRADE, IN CLASS C SCHOOLS

	Addition	Subtraction	Multiplication	Division
A	94.2%	88.3%	100%	100%
B	82.4%	65.4%	93%	91.7%
C	79.6%	64.3%	86.1%	87.5%
D	61.1%	47.7%	62.5%	74.5%
F	65.9%	59.1%	81.7%	87.5%

Retention of Boys and Girls Compared

The number of students who cooperated in the examination was almost equally divided between boys and girls. The total number was two hundred and ninety boys and three hundred and twelve girls.

The boys correctly answered one thousand eight hundred and fifty-six of the two thousand three hundred and twenty addition problems or seventy-nine and eight-tenths percent.

The girls correctly answered one thousand nine hundred and eighty-one of the two thousand four hundred and ninety-six problems or seventy-nine and three-tenths percent.

The boys correctly answered one thousand five hundred and seven of the subtraction problems or sixty-

four and eight-tenths percent.

The girls correctly answered one thousand six hundred and ninety of the subtraction problems or sixty-seven and three-tenths percent.

The boys correctly answered two thousand and twenty of the multiplication problems or eighty-seven and one-tenth percent.

The girls correctly answered two thousand one hundred and sixty-three multiplication problems or eighty-six and five-tenths percent.

The boys correctly answered two thousand one hundred and twenty-three of the division problems or ninety-one and three-tenths percent.

The girls correctly answered two thousand two hundred and sixty of the division problems or ninety and four-tenths percent.

TABLE XII

PERCENTAGE OF CORRECT ANSWERS FOR BOYS AND GIRLS

	Addition	Subtraction	Multiplication	Division
Boys	79.8%	64.8%	87.1%	91.3%
Girls	79.3%	67.3%	86.5%	90.4%

CHAPTER III

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The various comparisons that were made point to the conclusion that there is a lack of knowledge regarding the manipulation of positive and negative numbers. This is definitely noticeable in the solving of subtraction problems. The rule governing subtraction problems, "change the sign of the subtrahend and proceed the same as in addition", appears to be the difficulty that is encountered. Multiplication and division problems are correctly solved in eighty-five or more precent of the cases. Approximately eighty percent of the addition problems were correctly solved.

There is little or no evidence which indicates that one type or classification of school, such as A, B, and C, as listed in the Kansas Directory, is better than another in regard to teaching of positive and negative number in algebra.

Psychologists in the main point out that there is a tendency for students with a high I. Q. to retain more than those students with a low I. Q. However, in this study, two students with high I. Q.'s failed to answer a

large percentage correctly, which may be due to a lack of interest in algebra, at least in that part represented by positive and negative number. Also, students, with an I. Q. below ninety correctly solved ninety-two and one-tenth percent of the division problems involving two negative numbers, whereas, only eighty-five and three-tenths percent were correctly solved by students with an I. Q. between ninety and one hundred and ten. Possible reasons for this are that the students with a low I. Q. might have had instructors that dwelt more on this particular phase, or the students might have been more interested in this particular section. On the other hand, one student with a low I. Q. answered every problem correctly. This student undoubtedly had a great interest in algebra, at least regarding positive and negative number.

The amount of knowledge retained, concerning the manipulation of positive and negative number, was in direct proportion to the scholastic grade received in algebra. This generalization holds true on the average of students. However, there are exceptions. Two students, who had received an "F" grade in algebra, correctly solved ninety percent of the problems. The "F" grade undoubtedly was based on something other than an understanding of positive and negative number.

Neither the boys nor the girls showed any indication, on the average, of outclassing the other in this phase of algebra. However, in addition, multiplication and division boys showed a very slight advantage. On the other hand, the girls were, on the average, two and five-tenths percent better than the boys in subtraction problems.

Recommendations

On the basis of this study the writer feels justified in making the following recommendations which probably are of value to the teaching profession: first, clearly more time should be devoted to the study of positive and negative number particularly of problems in subtraction; second, greater stress should be placed on a few simple rules such as the following: in addition problems if the signs are the same add the problem and use the common sign in the answer, if the signs are unlike find the difference and use the sign of the larger; in subtraction problems change the sign of the subtrahend and proceed the same as in addition; in multiplication and division problems if the signs are alike the answer is always positive, if the signs are unlike the answer is always negative; third, students with a high I. Q., and who learn readily, might well be encouraged to make a further study in mathematics through the use of the principle of acceleration.

APPENDIX

The writer wishes to take this opportunity of giving special recognition to the schools which cooperated in the operation of this test.

Antrim
Arnold
Bazine
Brownell
Bunkerhill
Circleville
Codell
Damar
Dennison
Elwood
Ensign
Flush
Garfield
Grenola
Herndon
Hudson
Leoville
Lucas
Luray
Macksville
McCracken
Ness City
Netawaka
Otis
Palco
Park
Paxico
Radium
Ransom
Rozel
Sparks
Stockton
St. John
Summerfield
Utica
Victoria
Wakeeney
Waldo
Walnut
Weskan
White Cloud
Woodston
Whiting

Last Year's Final Grade in Algebra _____

udent's Name _____ Classification of school _____

Student's I. Q. _____

Signed Numbers

is is not a test in the usual sense of the term, it is merely a survey for the purpose of ascertaining the on of sophomores in high school algebra. The problems should be self-explanatory; but if you need an inter- on the person administering the test will make it for you.

dition as performed in algebra

.) $\begin{array}{r} -16 \\ 4 \\ \hline \end{array}$ (2.) $\begin{array}{r} 16 \\ -4 \\ \hline \end{array}$ (3.) $\begin{array}{r} 16 \\ 4 \\ \hline \end{array}$ (4.) $\begin{array}{r} -16 \\ -4 \\ \hline \end{array}$

btraction as performed in algebra

.) $\begin{array}{r} -16 \\ 4 \\ \hline \end{array}$ (6.) $\begin{array}{r} 16 \\ -4 \\ \hline \end{array}$ (7.) $\begin{array}{r} 16 \\ 4 \\ \hline \end{array}$ (8.) $\begin{array}{r} -16 \\ -4 \\ \hline \end{array}$

ultiplication as performed in algebra

.) $\begin{array}{r} -16 \\ 4 \\ \hline \end{array}$ (10.) $\begin{array}{r} 16 \\ -4 \\ \hline \end{array}$ (11.) $\begin{array}{r} 16 \\ 4 \\ \hline \end{array}$ (12.) $\begin{array}{r} -16 \\ -4 \\ \hline \end{array}$

ivision as performed in algebra

13) $\dfrac{-16}{4} =$ (14.) $\dfrac{16}{-4} =$ (15.) $\dfrac{16}{4} =$ (16.) $\dfrac{-16}{-4} =$

olve as indicated

17. $(-18) + (3) =$

18. $(18) + (-3) =$

19. $(18) + (3) =$

20. $(-18) + (-3) =$

21. $(-18) - (3) =$

22. $(18) - (-3) =$

23. $(18) - (3) =$

24. $(-18) - (-3) =$

25. $(-18) \times (3) =$

26. $(18) \times (-3) =$

27. $(18) \times (3) =$

28. $(-18) \times (-3) =$

29. $(-18) \div (3) =$

30. $(18) \div (-3) =$

31. $(18) \div (3) =$

32. $(-18) \div (-3) =$

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