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A Research In Sixth Grade Children's Solution of Verbal Arithmetic Problems

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A RESEARCH IN SIXTH GRADE CHILDREN'S SOLUTION

OF VERBAL ARITHMETIC PROBLEMS

being

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To

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A RESEARCH IN SIXTH GRADE CHILDREN'S SOLUTION OF VERBAL ARITHMETIC PROBLEMS

CHAPTER I

INTRODUCTION

1. The Problem

Arithmetic has probably contributed more to non-promotion of pupils in grades above the first than any other subject of the curriculum. The verbal reasoning problem in arithmetic has not fared well in the history of elementary instruction. Perhaps computation has been easier to teach, at least it has fared better. The improvement in accuracy of computation seems to have produced little, if any, improvement in the accuracy of arithmetical reasoning. The evidence does not seem to indicate that there is such a community of function between computation and reasoning, in arithmetic, that improvement in the one operation necessarily involves improvement in the other. The inference should not be made from such a statement that there is not a rather high correlation between computation and ability to solve verbal arithmetic problems. The statement does imply that ability to compute does not insure ability to reason. It is debatable whether verbal problems offer children training in thinking but it is generally agreed that such problems do offer them opportunities for thinking. Is it correct to assume that the responses made by pupils in their efforts to solve verbal problems are the result of critical thinking?

The purpose of this research in sixth grade children's solution of verbal problems in arithmetic is to investigate the mental processes that lie back of pupils' answers in arithmetic. That is, how do pupils solve verbal problems in arithmetic? The investigation involves several related problems such as; to what extent is the pupils' method of solution influenced by irrelevant data, cues, details, and numbers used in a problem?

The experimenter believed that an indication of how pupils solve verbal problems in arithmetic best could be obtained in three ways that are all related but not necessarily of equal importance. In this thesis the experimenter: (1) Studied the nature of pupils' responses to changes in the statement of a problem by means of a statistical analysis of a written test; (2) analyzed the pupils' written tests for further evidence as to the procedure followed by pupils in solving problems; (3) interviewed certain pupils, that is, gave them an oral test in which a more detailed analysis could be obtained as to the extent of their critical thinking.

2. The Definition of Terms

In this thesis, the word "problem" means a verbal arithmetic problem, or the process by which the operations to be performed are not specifically indicated but must be determined by the pupil from the context. Computation is used to refer to the handling of arithmetical processes, that is, the processes of adding, subtracting, multiplying, and dividing. Computation and fundamental operations are used as synonymous terms. Some problems are, in the last analysis, just examples for some pupils, while for others they are in every respect a problem. A problem today for a pupil may cease to be a problem for him tomorrow. These definitions are arbitary, and are used in the interest of clarity.

3. The Review of Previous Investigations

There have been many investigations relating to arithmetic, but as Buswell and Judd[1] have pointed out "the studies which make a concrete analysis of how children reason when dealing with arithmetic are few in number". Buswell continued the summary of educational investigations relating to arithmetic for succeeding years, in the Elementary School Journal each year up to the present time. Most of these investigations reported and annotated in the sum-

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1. G. T. Buswell and C. H. Judd, Summary of Educational Investigations Relating to Arithmetic. Chicago, The University of Chicago, 1925, p. 90.

maries deal with work other than analysis of how children solve problems. The conclusion that research dealing with the computational phase of arithmetic has received a disproportionate share of attention and that research in the reasoning processes of children in solving problems has been neglected, appears to be justified. The experimenter agrees with Kramer[2] that the reason so much stress has been put on computation is "not because we do not recognize the intrinsic value of reasoning, but because critical thinking in arithmetic apparently eludes quantitative study".

Morton[3], in commenting upon causes of difficulty in problem solving, had the following to say:

> The author has examined a vast amount of published material on this subject--thousands of pages--but has found few specific suggestions which can be relied upon to produce better results with pupils. Many of the suggestions are based upon experiments conducted with small numbers of pupils and some others are of the subjective or opinion type. Some of the results secured by different investigators fail to agree.

Monroe conducted an extensive study to determine how pupils solve problems in arithmetic. He obtained his data

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2. Grace A. Kramer, The Effect of Certain Factors in the Verbal Arithmetic Problem upon Children's Success in the Solution. Baltimore, Johns Hopkins Press, 1933, p. 27.

3. R. L. Morton, Teaching Arithmetic in the Elementary School, Book Two. Boston, Silver Burdett Company, 1938, p. 460.

by administering four tests of twelve problems in each test to pupils in the seventh grade with the inclusion of a few sixth and eighth grade classes. The tests were so constructed that it was possible to make comparison of pupil response to the effect of irrelevant data, abstract material, and technical terminology. Monroe concluded[4]:

> ...that a large percent of seventh-grade pupils do not reason in attempting to solve arithmetic problems. ...many of them appear to perform almost random calculations upon the numbers given. When they do solve a problem correctly, the response seems to be determined largely by habit. If the problem is stated in the terminology with which they are familiar and if there are no irrelevant data, their response is likely to be correct. On the other hand, if the problem is expressed in the unfamiliar terminology, or if it is a "new" one, relatively few pupils appear to attempt to reason. They either do not attempt to solve it or else give an incorrect solution.

Monroe's study has one limitation. The same group did not work all the problems but the data were treated as though they were from a single group. The sampling was large enough partially to overcome this limitation. The fact remains, however, that the variation of responses that were compared did not come from the same pupils but from four different groups.

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4. Walter S. Monroe, How Pupils Solve Problems in Arithmetic. (Investigations in the field of education, published by the University of Illinois. Prepared by the Bureau of Educational Research. University bulletin, no. 44, 1929.)

Bradford[5] tested a group of several hundred children in England. The tests were composed of problems impossible of solution, the thought of the author being that the extent to which pupils attempted to solve such problems was indicative of the absence of critical thinking. An example of one of the questions is, If Henry VIII had six wives, how many had Henry II? Bradford's conclusion was that since a high per cent worked out solutions for the problems that critical thinking was absent.

Kramer[6] made an elaborate study of the effect of interest, sentence form, style-language details, and vocabulary, upon sixth grade children's success in solving problems. The data were obtained by administering eight tests of sixteen problems each, to the 6B classes in the elementary schools in Baltimore. The arithmetical content of the tests employed paired problems in subject-matter of grade 5A, and were scored for principle. The concluding suggestions were: Not much can be accomplished merely through providing interesting problem material; there probably was no best sentence form; the style when brief, using only essential facts, resulted in more success; and, that pupils were more successful with problems stated in familiar vo-

5. Morton, op. cit., p. 467.

6. Kramer, op. cit., p. 48.

cabulary.

The practical conclusions were made that children did little reflective thinking, seldom verified their choice of operation, and seemed to respond more to the cue than to requirements of the problem.

An experiment was conducted by Bramhall[7] to determine the relative effectiveness of two types of problems in the improvement of the problem-solving ability of sixth grade pupils. No statistically significant difference between the conventional and imaginative type problems was found. A slight difference was found in favor of the imaginative problem. The suggestion was made that children do better when left to their own devices. In light of the data presented, this suggestion hardly seems justified.

An experiment was devised by Myers[8] to compare dry, concise, traditional problems with problems designed to stimulate vivid imagination. Six pairs of problems were given to 513 children in the fifth grade. An example of one of the pairs that has been quoted in several experiments is shown on the next page.

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7. Edwin W. Bramhall, "An Experimental Study of Two Arithmetic Problems". (In Journal of Experimental Education, vol. 18, September, 1939, p. 38.)

8. G. C. Myers, "Imagination in Arithmetic". (In Journal of Education, vol. 105, June 13, 1927, p. 662.)

Form 1. After traveling 160 miles a man has 4 gallons of gas left in his automobile. How many miles did he get to a gallon of gas if he bought 8 gallons on the way and had 6 gallons when he started?

Form 2. Last summer Agnes Purdy, her brother, Archie, and their parents took a trip in their Ford. Archie measured the gasoline when they started. "We have 8 gallons", he told his father. At the end of the day he found 4 gallons of gasoline in the tank. They had bought 6 gallons at a station on the way, and had traveled 160 miles. Agnes told her mother that they had made _____miles to a gallon that day.

Myers found the imaginative problem to be superior. Form 2 was correctly solved by 49 per cent, and Form 1 by 38 per cent. The findings are questionable because Form 1 and Form 2 were not written in the same chronological order, which may have made Form 1 more difficult.

White[9] found significant results supporting the thesis that experience in the situation involved affects the solving of a problem. Reference was made, by White, to the extensive study of Hydle and Clapp in which "they conclude that the nature of the situation as to familiarity has but little significance as a factor in problem solving".[10] White criticised this work because no attempt was made to discrimate between various types of wrong answers.

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9. Helen M. White, "Does Experience in the Situation Involved Affect the Solving of a Problem?" (In Education, vol. 54, April, 1934, p. 455.)

10. Ibid., p. 451.

A two year study of factors causing difficulty in problem solving was made by Washburne and Osborne[11]. The introduction included the remark, "pupils seem to have a way of doing the wrong thing, of simply juggling the numbers, that is most exasperating". As a result of their study these conclusions were drawn:

> ...that to train all children to go through a set, formal analysis of their problems is less effective than simply to give children many problems and to help each child with any special difficulty that he may encounter. Training in the seeing of analogies appears to be superior to analysis for the lower half; but merely giving many problems, without any special technique of analysis of the seeing of analogies, appears to be decidedly the most effective of all.

Washburne and Osborne refer to the working of many problems as the "individual method". They failed to find any relation between ability to make formal analysis and ability to solve problems. Several investigations have disagreed with these findings.

Clark and Vincent, Hanna, Otis, Newcomb, and Mitchell have reported studies relative to the use of analysis. Clark and Vincent[12] found the graphical analysis method

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11. Carleton W. Washburne and Raymond Osborne, "Solving Arithmetic Problems". (In _Elementary School Journal_, vol. 27, November and December, 1926, p. 303.)

12. J. R. Clark and E. L. Vincent, "A Comparison of Two Methods of Arithmetic Problem Analysis". (In _Mathematics Teacher_, vol. 18, April, 1925, p. 233.)

superior to the conventional method of analysis. Hanna[13] conducted a controlled experiment in which the dependencies method, a method similar to the graphical, produced better results than the conventional method of analysis but not better than the individual method. Otis[14] suggests the value of visual aids in analyzing problems. Although experimental evidence is lacking, his method is plausable. Newcomb[15] found that logical procedure in solving problems was superior to an undirected procedure. Mitchell[16] reports that detailed analytical questions asked by the teacher on problems helped in the solution by the children.

Although the studies dealing specifically with how children solve problems in arithmetic have been limited in number there have been numerous investigations showing correlations with ability in problem solving and certain

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13. Paul R. Hanna, "Methods of Arithmetic Problem Solving". (In Mathematics Teacher, vol. 23, November, 1930, p. 450.)

14. Arthus S. Otis, "The Visual Method of Solving Arithmetic Problems". (In Mathematics Teacher, vol. 21, December, 1928, p. 489.)

15. R. S. Newcomb, "Teaching Pupils How to Solve Problems in Arithmetic". (In Elementary School Journal, vol. 23, November, 1922, p. 189.)

16. Claude Mitchell, "Problem Analysis and Problem-Solving in Arithmetic". (In Elementary School Journal, vol. 32, February, 1932, p. 465.)

other factors. Morton[17], Brueckner[18], and Buckingham[19] have reported the following correlations:

	Morton	Brueckner	Buckingham
Verbal intelligence.......	.78	.50	.40
Non-verbal................	.52		
Computation...............	.70	.35	.59
Reading comprehension.....	.61		
Reading rate..............	.23		
Age in months.............	.34	....	-.20
Quantitative relationship.		.... .60	
Vocabulary................		.... .76	

The correlations found are typical of the many that could be cited. Authorities agree that correlation exists, but they are at variance with one another as to the degree of correlation.

When more exact information as to the pupil's method of work is needed the interview technique may be employed. If the experimenter secures the cooperation of the learner, systematic questioning will often reveal conditions that would otherwise be undetected. Winch reports a study that shows clearly the complex mental processes which occur in computation before a child is ready to write his

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17. Morton, op. cit., p. 454.

18. Leo J. Brueckner, "Diagnosis in Arithmetic". (In Whipple, Guy Montrose, Editor. Thirty-Fourth Yearbook of the Society for the Study of Education. Bloomington, Public School Publishing Company, [c1935], p. 273.

19. B. R. Buckingham, "Mathematical Ability as Related to General Intelligence". (In School Science and Mathematics, vol. 21, November, 1921, p. 20.)

answer on paper[20].

Cissy F___, aged 10 years, dealt with a sum containing several noughts in the minuend in this manner.

400,000
<u>   59</u> She said: (1) 9 from 0 I can't; go next door I can't; go next door I can't; go next door, I can't; go next door, I can't; go next door, take 1, leaves 3, and that makes that (pointing to the nought immediately to the right of the 4 in the minuend) 10. 9 from 0 I can't; go next door, I can't; go next door, I can't; go next door, I can't; go next door, take 1 from the 10 leaves 9, and makes that one (pointing to the nought in the second place from the 4) 10. 9 from 0 I can't; go next door I can't; go next door I can't; go next door, take 1 from the 10 leaves 9 and makes that (pointing to the third nought) a 10. 9 from 0 I can't; go next door, take 1, leaves that a 9 and makes this a 10; 9 from 10 leaves 1.
(2) 5 from 9 leaves 4. (3) 0 from 9 leaves 9.
(4) 0 from 9 leaves 9. (5) 0 from 9 leaves 9.
(6) 0 from 3 leaves 3.

Many studies using the interview technique have been reported in computation, but few have been reported of studies of how children solve verbal problems. Stevenson[21] has suggested that the form of a problem often decides what process to use. He related that a colored girl described her method as follows:

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20. G. T. Buswell, "A Critical Survey of Previous Research in Arithmetic". (In Whipple, Guy Montrose, <u>Editor</u>. <u>Twenty-Ninth Yearbook of the National Society for the Study of Education</u>. Bloomington, Public School Publishing Company, 1930, p. 465.)

21. P. R. Stevenson, "Difficulties in Problem Solving." (In <u>Journal of Educational Research</u>, vol. 11, February, 1925, p. 95.)

> Whenever they is lots of numbers, I adds, but when they is only two numbers with lots of parts [digits], I subtracts. But if they is just two numbers and one is littler than the other, I divides when they comes out even, and multiplies when they don't.

The interview technique was also used by Reed[22] in making a comparison between adult reasoning and the method employed by a child. An implication is given of the value of organization in working problems. The idea is expressed that although it may not be possible to teach certain insights to every pupil, if the pupil does not have them he cannot solve problems.

Dr. Thorndike has contributed much to the psychology of arithmetic. He contends[23] we used to think any problem

> ...that gave the mind a chance to reason would do; and pupils labored to find when the minute hand and hour hand would be together, or how many sheep a shepherd had if half of what he had plus ten was one third of twice what he had!

But Dr. Thorndike also maintains that it is a false inference[24]

> ...that most of the problems by which arithmetic learning is stimulated had better be external to arithmetic itself--problems about Noah's Ark or Easter Flowers or

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22. Homer B. Reed, Psychology of Elementary School Subjects. rev. ed. New York, Ginn & Company, [c1938], pp. 308-309.

23. E. L. Thorndike, The Psychology of Arithmetic. New York, The Macmillan Company, 1922, p. 20.

24. Ibid., p. 283.

the Merry Go Round or A Trip Down the Rhine. ...Outside interests should be kept in mind but it is folly to neglect the power, even for very young or for very stupid children, for the problem "How can I get the right answer?" Children do have intellectual interests. They do like to learn to add, subtract, multiply, and divide with integers, fractions, and decimals, and to work out quantitative relations.

Dr. Thorndike contends elsewhere[25] that

...Almost everything in arithmetic should be taught as a habit that has connections with habits already acquired and will work in an organization with other habits to come. The use of this organized hierarchy of habits to solve novel problems is reasoning.

Brueckner and Elwell[26] conducted an experiment in which they found that diagnosis based on single examples is inadequate.

...This investigation shows conclusively that errors in arithmetic processes made by superior as well as inferior workers are highly variable and that the mental processes involved in arithmetic cannot be readily explained on a simple mechanic l basis. If errors persisted steadily, or appeared in definite systems or patterns, the nature of the mental reactions of the learner might be quite readily analyzed. As it is, owing to the complicated nature of the learning process, we must admit the relative inadequacy of our present techniques of analysis and diagnosis.[27]

It is evident that scientific evidence of how children solve problems in arithmetic is lacking. The useful-

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25. Ibid., p. 194.

26. L. J. Brueckner and M. Elwell, "Reliability of Diagnosis of Errors in Multiplication of Fractions". (In Journal of Educational Research, vol. 26, November, 1932, pp. 175-185.)

27. Brueckner, op. cit., p. 291.

ness of analysis of errors, or the study of pupil reactions to problems depends upon the details to which the study has been carried. For example, it is obvious that to conclude a certain per cent of errors is due to total failure to comprehend the problem, needs further analysis. It is of some value to know that a child does not comprehend a problem, but it is of far more value to know the probable reasons why children fail to comprehend problems. Finding how and why mistakes are made in solving problems can not be detected solely from an analysis of written work, but require techniques that are more clinical in nature.

4. The Limitations

This investigation deals with the responses pupils make as the result of the instruction they have received, and therefore, the findings and generalizations made from the study must be considered in this light.

5. The Acknowledgments

Gratitude is expressed to Dr. Robert F. McGrath under whose immediate direction this study was conducted. Gratitude is also expressed to Dr. Floyd B. Streeter and Dr. Donald M. Johnson as well as to the numerous school administrators and teachers who helped to make this study possible.

CHAPTER II

THE DATA AND THEIR TREATMENT

1. The Experimental Tests

The written test for this study was composed of two equal parts, Tests A and B, making a total of 40 problems. The test was designed in companion problems in order to compare the effect of changing certain factors in a problem. The companion problems were exactly alike in difficulty of computation and method of solution except for one factor. Each child worked the paired problems. The hypothesis of the experimenter was that if a significant difference were found in the solution of the problems compared, it could be accounted for by the experimental factor since the subjects remained constant and only the conditions were varied by the experimental factor.

The problems compared, with a minor exception, have the same number. For example, problem 3 in Test A is compared with problem 3 in Test B. The factors isolated and problems compared will be explained more fully in the appropriate place. In general, the purpose of isolating certain factors in the paired problems is to determine to what extent the pupils' method of solution is influenced by cues, irrelevant material, details, and the type of numbers used in the problems.

Brueckner[1] contends that the basis of the norms on standardized tests in problem solving is open to question since the scores are usually expressed in the number of answers correct. He has shown that from 20 to 40 per cent of incorrect solutions are due to errors in computation. Hence a pupil's score is low because of his inability to compute accurately, and not because of his inability to reason out the method of solving a problem.

It seems reasonable, therefore, that the difficulty of the computations should be reduced to a minimum. This policy was followed in constructing this test. In no case was any computation called for in the solution of a problem in the written test that was beyond fifth grade level. The experimenter assumed that if the pupil became too involved in the computation it would not be a valid reasoning test. The problems were scored for correct answer, rather than principle. The plan was followed since computation in every case was relatively simple, and because scoring on this basis was more objective than scoring for principle. This plan was also followed because the experimenter believed that ability to recognize the probable answer and checking its reasonableness is an essential

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1. Brueckner, _op_. _cit_., p. 293.

part of reasoning in arithmetic. For example, a pupil in solving a problem finds that a car, which will run 15.5 miles on one gallon of gasoline, will run 1550 miles on 10 gallons of gasoline. The pupil is hardly entitled to have the problem marked correct in principle because he multiplied.

Considerable research was done by the experimenter in an effort to make the written test valid. The nature of the test made it impossible to obtain validity coefficient with an outside criteria. There was no test available that would measure the particular factors under consideration in this study. However, other reasoning tests for this grade level were studied, text books were consulted, and related studies were of considerable value, particularly those of Kramer[2] and Monroe[3]. Many of the problems were selected or adapted from other tests. The experimenter's interest and his six years experience in teaching arithmetic did not insure his construction of a valid test but it may have helped to make the research in the field more significant to him.

In many cases the steps used in solving problems are taken mentally and there is little objective record avail-

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2. Kramer, op. cit., pp. 91-96.

3. Monroe, op. cit., pp. 20-25.

able to give any sort of clue as to the thought processes that are used. Test C, an oral test, was given in an interview with the pupils, to get additional evidence as to the procedure pupils follow in solving a problem. The first ten problems from Test C were adapted or taken directly from Form 5 of the Army Group Examination Alpha. They are graded as to difficulty and are more difficult than the problems of Tests A and B. An additional two problems were included. These two problems are impossible of a correct solution. They were given to obtain additional evidence as to the extent of the pupil's critical thinking and the procedure he uses in solving problems.

2. The Experimental Group

The experimental group was comprised of 518 sixth grade pupils. The pupils tested were in the following cities in Kansas: Pratt, Haven, Russell, Norton, Ellis, Kinsley, Stockton, Hays, and Rural Districts 12 and 59 in Ellis County. The Oral Test was given to twenty-three pupils in the four different sixth grade classes in Hays, and in District 59. Each of these five classes was taught by a different teacher. Those taking the oral test had first taken Tests A and B.

The schools selected insure at least a fair representative sampling of the school population at this level. The schools were scattered, various types of communities were represented, small schools and large schools were tested, and pupils of 15 different teachers were represented.

The experimenter did not administer intelligence tests but several of the schools tested had data on the intelligence of their pupils. The evidence would indicate that the group as a whole would have a mean I.Q. that is normal for sixth grade pupils in Kansas.

The experimental group had a mean age of 11.93 years at the end of March. This mean age is the typical age to be expected since the average sixth grade pupil becomes twelve years old before the school year is completed. There is reason to believe that the sampling is representative of typical Kansas sixth grade children.

3. The Administration of the Tests

The tests were administered on two consecutive days in the last two weeks of March, 1942. The effect of practice and related problems had to be eliminated as much as possible since the problems were paired. Therefore, it was necessary to devise a scheme so that half of the experimental group worked Test A the first day and half of the experimental group worked Test B the first day. Likewise, so that half of the group worked Test A the second day and half of the group worked Test B the second day.

Such a scheme was devised. For example, Kinsley had two sixth grade classes.

In class 1:

The girls took Test B and the boys Test A the first day; the girls took Test A and the boys Test B the second day.

In class 2:

The girls took Test A and the boys Test B the first day; the girls took Test B and the boys Test A the second day.

A similar plan was followed in the other schools.

This plan made it possible for each school to take half, or approximately so, of each test each day. In this way, if the instructions were not followed, and the tests were discussed before each pupil took both tests, the effect would be less disastrous since it would effect both Tests A and B alike.

The written test was either administered by the experimenter or administered under the direction of the administrative head of the school. In every instance, the written test was administered by one experienced in testing. The experimenter gave all the oral tests.

Tests A and B were printed on legal size, good quality paper. Plenty of room was allowed for computation so that all the pupil needed to supply was a pencil. Since the work was to be analyzed the pupils were instructed to show their work in the space provided and not use scrap paper. A convenient place was provided for the name, sex, age, answers, and for the data pertaining to the problems they liked or did not like.

4. The Reliability of the Test

The written test was given in two equal parts, Tests A and B. To determine the reliability of the test, the two halves were correlated. Figure 1 on the following page shows the calculation of the product-moment coefficient of correlation[4] between Tests A and B, with application of the Spearman-Brown formula[5] to determine the reliability coefficient of the whole written test. The reliability coefficient is .935 $\pm$.005, which is evidence that the test is a reliable instrument for measuring the abilities in question.

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4. Henry E. Garrett, Statistics in Psychology and Education. 2nd ed. New York, Longmans, Green & Co., 1939, p. 270.

5. Ibid., p. 315.

Score on Test B (X-Variable)

Score on Test A (Y-Variable)	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	f_y	d_y	fd_y	fd_y^2	d_xd_y
0																	1	2			3	-9	- 27	243	180
1																	4	2		1	7	-8	- 56	448	376
2															2	4	2	2	1		11	-7	- 77	539	434
3												1	1	1	3	1	6	2	1		16	-6	- 96	576	486
4												3	2	4	5	6	2				22	-5	-110	550	390
5											2	2	7	10	7	9	3	1			41	-4	-164	656	576
6									2	3	4	9	10	7	6	2					43	-3	-129	387	231
7									5	4	6	9	10	11	2	3	1				51	-2	-102	204	154
8								2	2	10	9	10	13	5	2	2					55	-1	- 55	55	49
9							1	4	7	8	16	16	6	2	2						62	0			
10					1	1	1	7	9	6	12	5	3	3							48	1	48	48	40
11					1	3	2	4	6	8	2	5	1								32	2	64	128	108
12				2			10	5	5			1									23	3	69	207	234
13					4	1	5	2	1	3		3	2								21	4	84	336	212
14		1		1	3	7	2	3	1	1											19	5	95	475	445
15			1	2	3	1	5	2		1	1										16	6	96	576	432
16		1	2	3	2	4		2													14	7	98	686	588
17			3	3	4	1		2													13	8	104	832	640
18		2	3	2	1	3															11	9	99	891	693
19		2	3	1																	6	10	60	600	490
20	3	1																			4	11	44	484	429
																					518		≤+46	8921	7187
f_x	3	7	12	14	19	21	26	33	38	44	52	64	55	43	29	27	19	9	2	1	518				
d_x	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9					
fd_x	30	63	96	98	114	105	104	99	76	44		-64	-110	-129	-116	-135	-114	-63	-16	-9	≤+73				
fd_x^2	300	567	768	686	684	525	416	297	152	44		64	220	387	464	675	684	441	128	81	7583				

$$r = \frac{\frac{\Sigma XY}{N} - C_x C_y}{\sigma x \quad \sigma y} = \frac{\frac{7187}{518} - \left(\frac{+73}{518}\right)\left(\frac{+46}{518}\right)}{(3.80)\ (4.15)}$$

$$r = .878 \quad \pm.010$$

$$r_{nn} = \frac{nr_{1I}}{1 - (n + 1)\ r_{1I}} = \frac{2 \times .878}{1 + .878} \qquad r = .935 \quad \pm.005$$

Fig. 1. Calculation of the Reliability Coefficient Between the Two Halves of the Written Test Given to 518 Sixth Grade Pupils. Scores Represent Number of Answers Wrong.

CHAPTER III

THE FINDINGS OF THIS STUDY

1. The Effect of Irrelevant Data

The purpose of Table I is to give the findings relative to the effect upon the pupils' solutions when irrelevant data are introduced into problems. The table suggests that the pupils do not discriminate between relevant and irrelevant data.

Table I

Comparison of Pupils' Responses When Problems Contained Only Relevant Data, and When Problems Contained Irrelevant Data, with Other Factors Remaining Constant

Problems compared		Correct answers (518 pupils)		Per cent correct		
R	I	R	I	R	I	Diff.
A-3	B-3	324	34	62.5	6.5	+56.0
B-5	A-5	446	425	86.1	82.0	+4.1
B-15	A-15	103	105	19.8	20.2	-0.4
A-20	B-20	102	69	19.6	13.3	+6.3
Summary		975	633	47.0	30.5	+16.5

Note: This table is to be read as follows: Problem 3 in Test A, which contained only relevant data (R), was compared with problem 3 in Test B, which contained irrelevant data (I). Problem A-3 was worked correctly by 324, and problem B-3 by 34, of the 518 pupils. Problem A-3 was worked correctly by 62.5 per cent, problem B-3 by 6.5 per cent, a difference of 56 per cent.

Apparently the experimental factor, irrelevant data, has had an effect on the experimental group as the summary in Table I shows a difference. Is this difference reliable, that is, is it significant?

To answer this question the formula[1] for calculating the significance between obtained means was used, after first computing the necessary data needed for the formula.

$$\sigma D = \sqrt{\sigma^2_{M_1} + \sigma^2_{M_2}} = .12 \quad \text{Diff.} = .66 \pm .12 \quad D/\sigma_D = 5.5$$

The obtained difference is significant since the "critical ratio" (5.5) is greater than three.[2] This finding supports the thesis that pupils, in solving a problem,

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1. Garrett, op. cit., pp. 211-218.

2. In this study the "single group" took both Tests A and B, therefore, had the "critical ratio" been less than three, it would have been necessary to use the longer formula σD, which accounts for correlated means. However, since the use of this longer formula always tends to make the standard error of the difference smaller and the "critical ratio" larger, it is a measure of safety to use the simpler formula above.

Practically the same "critical ratio" (5.6) was obtained by using the formula for the standard error of the difference between two uncorrelated percentages.

$$\sigma D_p = \sqrt{\sigma^2_{p_1} + \sigma^2_{p_2}} = .029 \quad \text{Diff.} = .165 \pm .029 \quad D/\sigma_{D_p} = 5.6$$

This formula is more convenient to use but confidence can be put in the result only when the "critical ratio" is greater than three when the "single group" method is employed. In this thesis, since it is sometimes necessary to account for correlation, the formula for the σD will be used. (See Garrett, pp. 228-229).

do not disregard irrelevancies, but tend to compute with whatever quantities they find in a problem with little regard for the purpose of the quantities.

Comparison of problems 15-A and 15-B in Table I indicate a slight inconsistency in the findings. Examination of the test papers offer an explanation for this variation. Most of the pupils missed these two simple problems because they did not observe the word "left". The pupils who did observe this word, evidently disregarded the irrelevant material.

2. The Effect of Details

The purpose of Table II is to show the findings relative to the effect upon the pupils' solutions when problems are written in abstract form or without details, and when they are written in concrete form or with details.

The table suggests that the pupils work one type of problem about as well as another, but the variations that do exist are in favor of the problems written with details or in concrete form. In only one set of paired problems was the percentage of difference large. An analysis of the two problems, B-9 and A-9, offers a possible reason. The additional words, "Walter paid", in problem A-9 may have helped make it easier to work.

Table II

Comparison of Pupils' Responses When Problems Were Written in Concrete Form or with Details, and When Problems Were Written in Abstract Form or without Details, with Other Factors Remaining Constant

Problems compared		Correct answers (518 pupils)		Per cent correct		
C	A	C	A	C	A	Diff.
A-4	B-4	394	384	76.0	74.1	+1.9
B-9	A-9	383	332	73.9	64.0	+9.9
B-11	A-11	209	222	40.3	42.8	-2.5
A-14	B-14	433	438	83.5	84.5	-1.0
Summary		1419	1376	68.4	66.4	+2.0

To find if the difference shown in the summary of Table II is significant, the formula[3] for calculating the significance between obtained correlated means was used, after first computing the necessary data for the formula.

$$\sigma D = \sqrt{\sigma^2_{M_1} + \sigma^2_{M_2} - 2r_{12}\sigma_{M_1}\sigma_{M_2}} = .056 \quad \text{Diff.} = .09 \pm .056$$

The "critical ratio" is 1.6, therefore, the findings are suggestive but not significant.[4] This is to be expected since, as Table II shows, in all but the one case

3. *Ibid.*, p. 218.

4. The "critical ratio" was .7 when computed by the shorter formula for the σ D. (See the first paragraph in the second footnote on page 25.)

already cited, the differences in percentages are small, two of the signs are plus and two are minus.

The experimenter believes he is justified in concluding that pupils are about as successful in working abstract problems or problems without details, as they are in working concrete problems or problems with details.

3. The Effect of Cues

The purpose of Table III is to give the data relative to the effect upon the pupils' solution of introducing certain cues in the statement of a problem.

Kramer, in a keen analysis of children's work in arithmetic, suggests that children

> ...frequently made their response neither to the total situation presented in the problem nor to an essential element or fact given in the statement, but to some familiar expression accepted or seized upon as a cue.[5]

The findings shown in Table III tend to support her thesis.[6]

The formula for calculating the significance between obtained correlated means was applied, after computing the necessary data for the formula.

$$\sigma D = .113 \qquad \text{Diff.} = .384 \pm .113$$

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5. Kramer, op. cit., p. 68.

6. Kramer did not specifically attempt to measure the effect of cues in her experiment because it was not her immediate problem.

The "critical ratio" being 3.3, the obtained difference is significant.

Table III

Comparison of Pupils' Responses When Certain Cues Are Introduced or Changed, with Other Factors Remaining Constant

Problems compared		Correct answers (518 pupils)		Per cent correct		Diff.
A-1	B-1	449	423	86.6	81.6	+5.0
A-2	B-2	442	424	85.3	81.8	+3.5
B-6	A-6	151	99	29.1	19.1	+10.1
B-7	A-7	234	220	45.1	42.4	+2.7
A-8	B-8	343	353	66.2	68.1	-1.9
A-10	B-10	380	317	73.3	61.1	+12.2
A-16	B-16	324	315	62.5	60.8	+1.7
B-18	A-18	183	154	35.3	29.7	+5.6
Summary		2506	2305	60.4	55.6	+4.8

The experimenter wishes to point out that the effect of cues is difficult to analyze and to measure because they are difficult to isolate. It is possible that it may be some factor, other than the cue, that has caused the difference in the pupils' solutions. Analysis of the pupils' papers and the oral interviews, however, tend to support the findings that pupils do tend to make unthinking responses when they come upon familiar cues. They "apparently reason" correctly when a certain cue indicates to divide but when presented with a problem that requires the "same reasoning" they may multiply in the absence of the familiar cue.

4. The Effect of Fractions

The findings relative to the effect upon the pupils' solutions of problems, when fractions are introduced in place of integers, are shown in Table IV. Each of the paired problems require essentially the same reasoning, but the pupils do not seem to think in fractions.

Table IV

Comparison of Pupils' Responses When Fractions Are Used Instead of Integers, with Other Factors Remaining Constant

Problems compared		Correct answers (518 pupils)		Per cent correct		
I	F	I	F	I	F	Diff.
B-12	A-13	433	349	83.5	67.3	16.2
A-12	B-13	127	107	24.5	20.6	3.9
A-17	B-17	349	100	67.3	19.3	48.0
Summary		909	556	58.4	35.7	22.7

The differences between the percentages is significant.

$\sigma D_p = .029 \qquad D = .227 \pm .029$

The "critical ratio" is 7.8, disregarding correlation.

The pupils seem to have pre-conceived notions as to how a problem should be worked before it is carefully read. That is, the type of quantities employed in the problem seem to become a cue to the pupil. They do not analyze the total situation before starting to work the problem.

Two problems requiring division are cited from the tests to illustrate the point.

17-A. The girls can make a doll house in 48 hours. They are working on it 2 hours a day. How many days will it take to finish the doll house?

17-B. The boys can build a boat in 36 hours. They are working on it 3/4 of an hour a day. How many days will it take to finish the boat?

The first problem was correctly solved by 67.3 per cent. The second problem also required division but the "3/4" in the problem evidently became a cue to multiply, at least an analysis of the papers revealed that about 70 per cent of the pupils multiplied. Only 19.3 per cent solved it correctly. An inspection of the two problems will reveal that they are essentially the same, except in one problem a fraction has been used in place of an integer.

Table V illustrates that the form of the question being asked is of minor importance to the pupil. When one problem appears to be the same as another, and the quantities used are similar, the same process is used by the pupil, even though the difference in the form of the question requires that a different process be used.

Table V

Comparison of Pupils' Responses When Given the Same Data But Form of Question Changed so as to Require a Different Method of Solution

Problems compared		Correct answers (518 pupils)		Per cent correct		
M	D	M	D	M	D	Diff.
B-12	A-12	433	127	83.5	24.5	59.0
A-13	B-13	349	107	67.3	20.6	46.7
Summary		782	234	75.4	22.5	52.9

Note: The different methods of solution required are multiplication (M) and division (D).

Two problems are cited from the test material.

13-A. In drilling his oats, a farmer plans to use 3/4 bu. of seed oats per acre. How many bushels will it take to plant 24 acres?

13-B. In drilling his wheat, a farmer plans to use 3/4 bu. of seed wheat per acre. How many acres will 24 bushels plant?

Most of the pupils multiplied in both problems in attempting to solve them, even though the latter required that division be used. "When in doubt, multiply" seems to be the guiding factor.

The results shown in Tables IV and V support the thesis that pupils are unduly influenced by simple fractions. There seems to be little transfer of knowledge in solving problems in which integers are used and in solving nearly identical problems in which simple fractions are employed.

5. The Supplementary Findings

a. The Preference for Certain Problems

When Tests A and B were formulated, two questions were included in each test pertaining to the pupils' likes and dislikes of the problems included in the tests. The questions were:

Which two of the 20 problems in this test did you like best?
Which two of the 20 problems in this test did you not like?

Table VI indicates the pupils' selections of the first eight of the forty problems.

Table VI

Problems Liked and the Number of Pupils Liking Them, with Per Cent Correctly Solved; and Problems Not Liked and Number of Pupils Not Liking Them, with Per Cent Correctly Solved.

Problems liked best			Problems not liked		
Problem number	Number pupils	Per cent correct	Problem number	Number pupils	Per cent correct
B-1	191	81.6	B-20	215	13.3
A-1	171	86.8	B-11	159	40.3
B-12	113	83.5	A-20	142	19.6
A-2	95	85.3	A-11	123	42.8
B-2	88	81.8	B-19	111	8.7
A-14	71	83.5	B-3	73	6.5
A-5	69	82.0	A-18	70	29.7
B-15	63	19.8	A-15	70	20.2

The forty problems were ranked according to the number of times they were disliked and missed in order to obtain the coefficient of correlation by the rank difference method[7] between problems disliked and problems missed.

$$\rho = 1 - \frac{6 \Sigma D^2}{N(N^2-1)} = .830 \pm .03 \qquad r = .842$$

The relatively high correlation indicates that pupils cannot work the problems they dislike.

The correlation coefficient between the problems liked best, and the problems correctly worked the greatest number of times, was obtained by the same method.

$$\rho = .477 \pm .08 \qquad r = .494$$

The relatively low correlation would indicate that there may be some relation between liking a problem and the ability to solve it, but it is not a very dependable guide.

Dr. Myers' oft quoted "imaginative problem" (B-11)[8] did not fare well in this study. It was the second highest of the forty problems disliked. When Myers' companion problem, designed to be without details and less imaginative, was rearranged and presented in the same chronological order as his imaginative problem, it (A-11) was worked correctly in this study by 42.8 per cent of the pupils as

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7. Garrett, <u>op</u>. <u>cit</u>., pp. 362-363.

8. See pages 7 and 8 in this study.

compared with 40.3 per cent for his highly imaginative problem (B-11). Klapper[9] suggested that if the problem were rearranged this might be the case. Myers' imaginative problem as compared with the dry, concise, traditional sort, was neither more interesting to the pupils nor was it more condusive to correct arithmetical reasoning, even though it was designed for the mere enjoyment of reading.

Perhaps it should be pointed out that the comments on Myers' imaginative problems are incidental to this study, and it is not to be infered that the experimenter has disproved the thesis held by Dr. Myers. He certainly is to be commended for his efforts in making arithmetic more interesting. The point the experimenter wishes to make is that any type of problem is of value only in so far as it contributes to correct arithmetical reasoning, that is, if it helps the child to think.

To summarize this particular section of the supplementary investigation, the findings tend to support the thesis that ability to work a problem does not insure that pupils will like it, but inability to work a problem does seem to be a fair indication that the pupils will not like it.

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9. Paul Klapper, The Teaching of Arithmetic. New York, D. Appleton-Century Company, Incorporated, [c1934], p. 439.

b. The Comparison of Sex Differences

The differences found between the sexes in this study are negligible. The boys excelled the girls .02 per cent in the solution of the verbal problems on the written test. Table VII indicates that the achievement of the two sexes are nearly equal.

Table VII

Comparison of Sex Differences in Achievement as Measured by the Forty Problems in Tests A and B.

Number of:		Number problems	Number correct	Per cent correct	Mean correct	S. D.
Boys	255	10,200	5298	51.94	20.78	7.56
Girls	263	10,520	5462	51.92	20.77	7.53

Note: The standard error of the difference between the two uncorrelated means is .657, Diff. = .01 ±.657, and .015 is the "critical ratio".

There is no significant difference in the ability of boys and girls to solve verbal problems. The chances are even that either group could excel the other. This suggests that there would be little, if any, justification for expecting one sex to excel the other in reasoning in arithmetic.

c. The Responses to Problems Impossible of Solution

Two problems were presented in Test C, the oral test, that were impossible of solution. The responses by the pupils to these two problems suggest that pupils do not deem it necessary to understand a problem before working out a solution. The problems were:

11-C. A boy is five years old and his father is 35 years old. If his uncle is 40 years, how old will his cousin be?

12-C. If a fencing costs 80 cents a foot, how much will it cost to put a fence around a garden 40 feet long?

Most of the pupils obtained answers for the two problems without noticing that they could not be solved even though in the interview they were asked if their answers were reasonable, and were asked to check their work. Only 30.4 per cent of the pupils suspected anything wrong with problem 11-C, and only 13.0 per cent observed that essential data were needed in problem 12-C before it could be solved rationally. The answer for the latter problem was obtained by 82.6 per cent of the pupils by simply multiplying 80¢ x 40. Superior, average, and below average pupils were included in the group which obtained such answers to the two problems cited.

Because of the small number of subjects included in the oral interviews these findings should not be considered conclusive but they are suggestive. Pupils apparently do not analyze the total situation before obtaining an answer to a problem, but they tend to compute with whatever quantities they find in a problem with little regard for re-

CHAPTER IV

THE ORAL INTERVIEWS

The purpose of this chapter is to give in some detail the actual responses pupils make when presented with verbal problems in arithmetic. The data were obtained by interviewing pupils, that is, by giving them tests in which they talked as they worked their problems.[1] As has been indicated elsewhere, the oral tests were supplementary to the written test and the findings from the data have been incorporated in the previous chapter. The reader could, therefore, omit this chapter without losing the continuity of this study but in so doing one would miss, the experimenter believes, some essential aspects of how children solve problems that can not be gained from a statistical analysis of written work.

A short history of the pupils under consideration is given in Table VIII which may help to make their responses

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1. An endeavor was made to select a representative group of the 518 pupils in the experimental group. They were selected on the basis of school records, scores made on written Tests A and B, and on teacher's judgment and knowledge of pupils' ability. Since the purpose of the oral test was only to get additional evidence as to the procedure pupils follow in solving problems and the extent of their critical thinking, it was thought that this method of selecting the group would be satisfactory. Pupils of extremely low mental ability were not interviewed.

more significant to the reader.

Table VIII

Concise Story of Certain Pupils' School Records, Percentile Rank on Tests A and B, and Home Background

Case	School records Arith.	General	Percentile Rand on Tests A and B	Home Background
F	A	A	95	Good
G	D	D	10	Poor
H	C-	C	60	Fair
J	B	B	65	Fair
K	C	C	25	Good
L	A-	A-	85	Superior
M	A-	A-	98	Excellent
N	C-	C	40	Superior
O	D	D	10	Poor
P	C	C	65	Fair
Q	C	C	30	Fair
R	B	B	45	Superior
S	C	C	40	Fair
T	B	B	75	Excellent
U	A	B	99	Poor
V	D	C	30	Poor
W	C	B	45	Excellent
X	A	B	70	Good
Y	C	C	15	Fair
Z	A	B	95	Good

Note: Considerable effort was made to give an accurate picture in this table of the pupils under consideration but even at the best, a considerable amount of it is based on the judgment of the experimenter and the teachers. Even with this limitation and the fact that it is so general, it is hoped it may be of some value to the reader.

All those pupils taking the oral test do not appear in Table VIII but only the cases cited in the chapter. Again the judgment of the experimenter entered in as to what cases to select to be representative.

Four items should be noted relative to the pupils' responses: First, they often use cumbersome methods in their computation which handicap their thinking; second, they usually have a purpose behind their work; third, their work is often not rational even though it may be purposeful; and fourth, the incorrect solutions cited are from all classes of pupils, poor, good, and superior. The evidence indicates that all classes of pupils make essentially the same errors, but the superior pupils make them less often.

All solutions pertaining to a certain problem are listed immediately following the problem. For example, under problem 9-C are the responses made by the various cases being cited. The problems are presented in the order of their difficulty to the group interviewed. The per cent of correct solutions by the group is indicated after each problem.[3]

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3. Each child was given time to check his work after completing the test. The experimenter read the problem while the pupil looked at his paper and checked his work. The pupil then repeated his answer. He was then asked if the answer seemed to be correct or reasonable. Because of so much repetition, this question and the pupil's reply are not recorded unless some significant remark was made. The absence of the question and reply indicate that the pupil thought his answer was reasonable.

In the cases reported, the material in parenthesis are the comments of the experimenter. The conversation of the experimenter is designated "E" and the conversation of the pupil "P". The particular pupil is designated by "Case F", "Case G", etc.

9-C. A ship has provisions to last her crew of 500 men 6 months. How long would it last 1,200 men? (4.3%)

Case F

P: "Divide 1200 by 500,

$$\begin{array}{r} 2 \\ 500\overline{)1200} \\ \underline{1000} \\ 200 \end{array}$$

, maybe that's wrong. That's got me stumped. Boy! More fun than playing chess!"

E: "Do you like to play chess?"

P: "Oh, I watch Daddy and ask questions but he won't answer. Mommy says it's 'cause Daddy has to think so deep. Divide 1200 by 500 -- I did that before but it didn't come out even so I put a decimal point after 1200 so it would come out even.

$$\begin{array}{r} 2.4 \\ 500\overline{)1200.00} \\ \underline{1000} \\ 200\ 0 \\ \underline{200\ 0} \end{array}$$

That answer is 2.4 months provisions will last."

Case J

P: "Subtract

$$\begin{array}{r} 1200 \\ \underline{500} \\ 700. \end{array}$$

Then I'd divide 700 by 6,

$$\begin{array}{r} 116 \\ 6\overline{)700} \\ \underline{6} \\ 10 \\ \underline{6} \\ 40 \end{array}$$

."

E: "Why did you divide 700 by 6 that first time?"

P: "To see how long it would last them for 6 months. I got 116 months and not that many months in a year.

$$\begin{array}{r} 2 \\ 500\overline{)1200} \\ \underline{1000} \\ 200 \end{array} \quad \frac{200}{500} = 2\frac{2}{5} \text{ months."}$$

Case Q

P: "Divide to see how many 500 in 1200.

$$\begin{array}{r} 2\frac{2}{5}. \\ 5\cancel{0}\cancel{0}\overline{)12\cancel{0}\cancel{0}} \\ \underline{10} \\ 2 \end{array}$$

$2\frac{2}{5}$ x 6 months $= 14\frac{2}{5}$ months."

E: "Why are you multiplying by 6?"

P: "Well, I got $2\frac{2}{5}$ here (pointing to answer above) and it was 6 months. I get $14\frac{2}{5}$ months."

Case R

P: "Six into 1200, $\begin{array}{r} 200 \\ 6\overline{)1200} \\ \underline{12} \\ 00 \end{array}$. That's silly -- couldn't last 200 -- that's silly. I'd divide 6 into 200 -- if it comes out even I'll keep it. $\begin{array}{r} 40 \\ 6\overline{)200} \end{array}$ -- couldn't be that. I am not keeping that so don't waste your paper. $\begin{array}{r} 2 \\ 500\overline{)1200} \\ \underline{1000} \\ 200 \end{array}$ -- No, that won't work.

E: "What do you mean, 'that won't work'?"

P: "It didn't come out even. Oh, now I am going to try something else. $\begin{array}{r} 1200 \\ \underline{500} \\ 700 \end{array}$, that's 700 men and for 500 it would be $6\overline{)700}$ -- wait, -- I don't know whether this is going to come out -- $\begin{array}{r} 116 \\ 6\overline{)700} \\ \underline{6} \\ 10 \\ \underline{6} \\ 50 \\ \underline{36} \\ 4 \end{array}$ -- it didn't work, either. $\begin{array}{r} 2 \\ 500\overline{)1200} \\ \underline{1000} \\ 200 \end{array}$ -- that's my answer."

Case U

P: "Six months -- that's 500. I can't work it, shall I go back to it?" (Pupil returned to this problem.)

"Gee! $\begin{array}{r} 83 \\ 6\overline{)500} \end{array}$, that isn't right. 500 men 6 months -- Now I know it! $\begin{array}{r} 3 \\ 2\overline{)6} \end{array}$ months, 1000 men. $\frac{2}{6} \times 500 = 1000 =$

$\begin{array}{r} 166\frac{2}{3} \\ 3\overline{)500} \end{array}$. Two-thirds! I can't have $\frac{2}{3}$ of a man, gee!

(Pupil changed his answer to 167 and wrote men.) 167 men for ---- $\begin{array}{r} 2 \\ 500\overline{)1000} \end{array}$ months for 1000 men -- I think that's right. No, there was 200 left -- gee, I did not do it right. It wouldn't last them more than -- $\begin{array}{r} 2\frac{2}{5} \\ 500\overline{)1200} \end{array}$ months."

$$\begin{array}{r} 2\frac{2}{5} \\ 500\overline{)1200} \\ \underline{1000} \\ \underline{2ØØ} \\ 5ØØ \end{array}$$

Case W

P: "What does that mean? You mean these same amount of provisions would last these 1,200 men? If it didn't have that 200 on there (pointing to 1,200) I'd get it."

E: "How would you do it if that 200 wasn't on there?"

P: "It would be 3 months. It would cut the provisions down one-half, so it would be $\frac{1}{2}$ less months."

E: "Now use 1,200."

P: "Yes! (Laughed.) That's what I am trying to do. Wouldn't it last 1,200 men $2\frac{1}{2}$ months, or would it?" (Pupil was right.)

E: "How did you get it?"

P: "I figured you'd ask me that. 200 off, would be $\frac{1}{2}$ off the 500 would do, or would it? It would be about $2\frac{1}{2}$ months, but I don't know."

Case X

P: "Well, I am going to divide 6 into 1,200. $\begin{array}{r} 200 \\ 6\overline{)1200} \end{array}$."

E: "What did you get?"

P: "I got 200 -- but --." (Implied that it was not right, and returned to it later.) $\begin{array}{r} 2 \\ 500\overline{)1200} \\ \underline{1100} \\ 200 \end{array}$ months and 200 days. $\begin{array}{r} 6\frac{2}{3} \\ 30\overline{)200} \\ \underline{120} \\ 20 \end{array}$ months. No --

it couldn't last them 6 months, because it only lasted 500 six months. I think it would last them only about 2 months -- but I don't know what to do with the 200."

Case Y

P: "It takes two 500's to make a 1000. It lasts 1000 men three months and 200 men about $\frac{2}{5}$ of a month.

$500\overline{)1000}^{\,2}$ months $3\frac{2}{5}$ months. $500\overline{)2000}^{\,\emptyset 4}$."

$$\begin{array}{r} 000 \\ \hline 2000 \\ \underline{2000} \end{array}$$

E: "What is that 4?"

P: "Wouldn't it be $\frac{4}{500}$ of a month? I was trying to get $\frac{2}{5}$ of a month but I couldn't get it."

5-C. If it takes 6 men 3 days to dig a 180-foot drain, how many men are needed to dig it in half a day? (13.0%)

Case F

P: "Gee! Divide 180 by 3, $3\overline{)180}^{\,60}$. Comes out 60 dig in one day. $2\underline{/60}$ 30, that would be half day -- that still comes out wrong."

E: "What do you mean 'comes out wrong'?"

P: "Because it doesn't tell how many it would take for a half day. I think I got it. $3\underline{/6}$ $1\underline{/2}$ 1, I divide by 2 because two parts in a day. No, still comes out wrong. One man to dig 180 foot drain in half day. Well, I can do it another way. 6 x 3 = 18 men to do it in one day, so you divide $2\underline{/18}$ 9 men in half day."

E: "Why divide 2 into 18?" (Pupil should have divided by one-half.)

P: "Because 2 parts in a day. I think 9 is more sensible than one."

E: "Why didn't you multiply 18 x 2?"

P: "Well, it would come out 36 that way and that wouldn't be a very reasonable answer." (Thirty-six is the correct answer.)

E: "Why?"

P: "Well, if only 6 men were working on it 3 days, gee, they would be slow workers."

E: "You say 18 men for 1 day?"

P: "Yes, 18 for one day."

E: "Now, if it took 18 men to dig it in one day, would it take less men to dig it in half a day?"

P: "No, it would take more. $\begin{array}{r} 18 \\ 2 \\ \hline 36 \end{array}$. Take 36. That seems like a lot."

Case H

P: "I'd just take 3 x 6. N-o-, half day -- just put 6 x 3 = 18."

E: "Is your answer reasonable?"

P: "I could do same way but I'd get same answer. I'll just leave it that way, except change 3 to 2."

E: "Why change 3 to 2?"

P: "Because half of 3 is 2, probably. I'll just leave it that way."

Case J

P: "I think I'll take 4 x 6 = 24 men to dig in half day. It would take 6 men for whole day."

E: "Where did you get your 4?"

P: "Well, 6, 6, and 6, are 18 men, 1 day. Need more men for half day. If they want to do it in half day I think you would add 6 more and make 24."

Case L

P: "Six men, 3 days. Let's see -- 3 into 6, I guess, goes -- 2 men for 1 day. $\begin{array}{r}3\overline{)6}\\2\end{array}$. How many men for $\frac{1}{2}$ day? $\begin{array}{r}1\\1\overline{/2}\end{array}$, 1 goes into 2, 1 man."

Case M

P: "Three days, 6
$\begin{array}{r}3\\2\overline{)18}\\9\end{array}$ men you would need. I think its wrong."

E: "Why do you think its wrong?"

P: "Because there isn't much to go on. You can usually find some facts to go on. Like this (pointed to problem 2) 66 miles -- you know how to start. This doesn't tell what is needed. It doesn't tell number of hours they might have worked."

Case T

P: "Oh, six men, -- 6 men, 3 days, then I would -- then half days -- it take -- it would take 6. If six half days -- so it took 6 x 6 = 36 men." (The pupil did all this in his head, writing down the figures, 6 x 6 = 36, after completing the problem.)

Case U

P: "Six men for 3 days. I don't know that one -- go on to the next one?"

E: "Yes, then you may come back to this one." (Pupil returned to this problem later.)

P: " $\begin{array}{r}60\\3\overline{)180}\end{array}$ days. $\begin{array}{r}10\\6\overline{)60}\end{array}$ ft. for each man. $\begin{array}{r}30\\2\overline{)60}\end{array}$ half days.

$\begin{array}{r}5\\6\overline{)30}\end{array}$ ft. per person. $\begin{array}{r}36\\5\overline{)180}\end{array}$ men. That isn't right."

E: "Why did you say it wasn't right?" (Pupil used wrong method but got the right answer.)

P: "Oh, yes, that's right."

Case Z

P: "Six x 3 = 18. $2\overline{)6}^{\,3.}$ $\begin{array}{r}18\\ \underline{3}\\ 21\end{array}$ -- I am not sure about that."

E: "Why not?"

P: "I didn't know half day. I could get it for 1 day."

E: "How many for 1 day?"

P: "For one day it would take 18." (A typical error, pupils can't think in fractions.)

7-C. A rectangular bin holds 400 cubic feet of lime. If the bin is 10 feet long and 5 feet wide, how deep is it? (17.3%)

Case F

P: "Ten times 5 = 50. I don't know whether that's right. I guess I'll let it be that way."

E: "What is the 50?"

P: "Gee whiz! You can't get chickens and mules and add them together. I thought one was yards and one feet but it isn't. I still think that's right."

E: "What is the 50?"

P: "Fifty feet deep it is."

E: "Is that answer reasonable?"

P: "I don't know because we haven't learned anything about cubic feet yet."

Case G

P: "Wouldn't you multiply 5 x 10 = 50? It would be 250 feet deep."

E: "How did you get it?"

P: "350."

E: "Can you show your work?"

P: "Bring down your $\begin{array}{r} 350 \\ \underline{50} \\ 400 \end{array}$."

E: "Where did you get that 350?" (Pupil really subtracted but couldn't explain how he got the 350.)

P: "Take 50 from 400."

E: "You said you could subtract. Can you show your work?"

P: "400 minus 50 -- ." (Pupil finally wrote $\begin{array}{r} 350. \\ \underline{50} \\ 400 \end{array}$)

Case J

P: "I think I'd change this 5 and 10 to cubic feet. $4\overline{)10}$ $2\frac{1}{2}$ $4\overline{)5}$ $1\frac{1}{4}$."

E: "Where did you get your 4?"

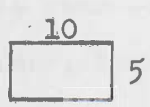
P: "When there's a square there's four sides. Then I think I'd add these -- $\begin{array}{r} 2\frac{1}{2} = 2 \\ \underline{1\frac{1}{4}} = 1 \\ 3\frac{3}{4} \end{array}$

$$\begin{array}{r} 106 \\ 3.75\overline{)400.00} \\ \underline{375} \\ 25\;00 \\ \underline{22\;50} \\ 2\;50 \end{array}$$ feet deep."

Case K

P: "$\begin{array}{r} 400 \\ \underline{10} \\ 4000 \\ \underline{5} \\ 20{,}000 \end{array}$ -- I got 20,000 feet deep. Yes, that's reasonable."

Case O

P: " 

$\begin{array}{r} 400 \\ 5 \\ \underline{10} \\ 415 \end{array}$."

E: "What are you doing there?"

P: "I am trying to add it, but it doesn't come out right."

E: "What do you mean?"

P: "I don't know. I think you would multiply. I just can't get that one."

Case T

P: "Ten feet long and 15 feet wide -- 10 x 10 = 20. $\begin{array}{r}10\\ \underline{20}\\ 30.\end{array}$"

E: "Where did you get 10?"

P: "I added 5 <u>times</u> 5."

E: "Where did you get 20?"

P: "Two times 10 = 20. I'm going to divide $30\overline{)400}$."

E: "Why?"

P: "To see how deep it is. $13\frac{1}{3}$."

$$\begin{array}{r} 13\frac{1}{3} \\ 30\overline{)400} \\ \underline{30} \\ 100 \\ \underline{90} \\ \frac{10}{30} = \frac{1}{3} \end{array}$$

E: "Thirteen and $\frac{1}{3}$ what?"

P: "$13\frac{1}{3}$ cubic feet deep."

Case W

P: (Whistled) "I never could get 'em in cubic feet. When you do this, do you put length x width and then x height? $\begin{array}{r}50\\ \underline{4}\\ 200\end{array}$ Say! It's supposed to be 400. That would be eight x fifty. Eight feet deep."

E: "Why did you multiply by 8?"

P: "Because 8 x 50 is 400 feet."

E: "Is your answer reasonable?"

P: "Yeah, I know that's right!"

10-C. If a submarine makes 8 miles an hour under water and 15 miles on the surface, how long will it take to cross a 100-mile channel, if it has to go two-fifths of the way under water? (21.7%)

Case H

P: "$\frac{2}{5}$ of $\frac{8}{1}$ - $\frac{16}{5}$ - $3\frac{1}{5}$ or 4 miles under water."

E: "What is that 4?"

P: "Under water."

E: "Then $\begin{array}{r} 100 \\ -\ 4 \\ \hline 96 \end{array}$ miles on the water?"

P: "Long? I got that mixed up. I probably add $\begin{array}{r} 8 \\ 15 \\ \hline 23 \end{array}$ together."

E: "Why did you add 8 and 15 together?"

P: "I don't know myself. 23 miles, though. $\begin{array}{r} 100 \\ 23 \\ \hline 77 \end{array}$ left. $\begin{array}{r} 23 \\ 2 \\ \hline 46 \end{array}$ $\begin{array}{r} 77 \\ 46 \\ \hline 31 \end{array}$, I'll put 4 into 31, $4\overline{)31}$."

E: "Why put 4 into 31?"

P: "Thirty-one miles left, $4\overline{)31}$ 70, and probably already gone 4 hours. The 60 minutes, $60\overline{)70}$ 10 minutes, probably left 60 minutes, $\begin{array}{r} 1 \\ 4 \\ 5 \end{array}$ hour hours, 10 seconds."

E: "Is that reasonable?"

P: "Sounds O. K."

Case J

P: "I think I'd divide 100 by $\frac{2}{5}$, $100 \div \frac{2}{5} = 40$ miles,

under water. Then subtract
$$\begin{array}{r} 100 \\ \underline{40} \\ 60 \end{array}$$ miles on surface.

Then 8 miles times
$$\begin{array}{r} 40 \\ \underline{8} \\ 320 \end{array}$$, 3 hrs. and 20 minutes, then (Notice how pupil took the liberty of changing 320 into 3 hrs. and 20 minutes.)

$$\begin{array}{r} 60 \\ \underline{16} \\ 300 \\ \underline{60} \\ 900 \end{array}$$, 9 hours.
$$\begin{array}{r} 3.20 \\ \underline{9.00} \\ 12.20 \end{array}$$, 12 hours and 20 minutes to go across the channel."

E: "Why 8 x 40?"

P: "Eight hours under water, so to find out how long, take 8 x 40, or 3 hours and 20 minutes."

Case M

P: "100 ÷ 2 = 250. $8\overline{)250}$ $31\frac{1}{4}$ hours."

E: "Where did you get 250?"

P: "I divided 100 by $\frac{2}{5}$. $\frac{100}{1} \times \frac{2}{5} = \frac{\overset{20}{\cancel{100}}}{1} \times \frac{2}{\cancel{5}} = 40$ $\quad 8\overline{)40}^{\,5}$."

Case N

P: "It will go $\frac{3}{5}$ of the way above. Each 5th would be 20 miles, so it would go 40 miles above water.
$$\begin{array}{r} 40 \\ \underline{8} \\ 320 \end{array}$$
hours, I think that's hours. 60 miles more to go.
$$\begin{array}{r} 60 \\ \underline{15} \\ 350 \\ \underline{60} \\ 1950 \end{array}$$ hours.
$$\begin{array}{r} \overline{)1950} \\ \underline{320} \\ 2270 \end{array}$$, I am adding hours. $24\overline{)2270}^{\,92\frac{1}{4}}$ days, plus nights and days. I don't know what I am doing, but I am doing it.
$$\begin{array}{r} 3\phantom{\frac{1}{4}} \\ 30\overline{)92\frac{1}{4}} \\ \underline{90\phantom{\frac{1}{4}}} \\ 2\frac{1}{4} \end{array}$$, No, that's right up there."

Case P

P: "
$$\begin{array}{r} 6\frac{2}{3} \\ 15\overline{)100} \\ \underline{90} \\ \frac{10}{15} = \frac{2}{3} \end{array}$$
."

E: "Why did you divide by 15?"

P: "It is 15 miles an hour on the surface. $\frac{2}{\cancel{5}} \times \frac{\overset{20}{\cancel{100}}}{1} = 40.$
$8\overline{)40}$ = 5 hours.
$$\begin{array}{r} 6\frac{2}{3} \\ \underline{5} \\ 11\frac{2}{3} \end{array}$$
hours."

E: "Why did you divide 8 into 40?"

P: "Because 40 is $\frac{2}{5}$ of the way."

E: "Why did you divide 15 into 100 then?"

P: "Oh, that is wrong -- 15 into 60 = 4.
$$\begin{array}{r} 5 \\ \underline{4} \\ 9 \end{array}$$
hours."

Case R

P: "Well, how many 15's and 8's would it take? I am going to try something but I don't think it's right.
$$\begin{array}{rr} 8 & 15 \\ 8 & 15 \\ 8 & 15 \\ \underline{8} & \underline{15} \\ 32 & 60 \end{array}$$
then
$$\begin{array}{r} 60 \\ \underline{32} \\ 90. \end{array}$$
"

E: "Why did you add just four 8's and four 15's?"

P: "Because I wanted it to come out 100, but it doesn't make 100, it's 92. Oh, I see,
$$\begin{array}{r} 40 \\ \underline{60} \\ 100. \end{array}$$
"

E: "Where did you get 40?"

P: "I added another 8.
$$\begin{array}{rl} 5 & (8's) \\ \underline{4} & (15's) \\ 9 & \end{array}$$
-- but I am just fooling around trying. But that isn't right 'cause I didn't use my $\frac{2}{5}$." (Pupil didn't know she had the right answer, 9.)

E: "What is the 5 + 4?"

P: "I don't know what they belong to. 40 under water, 60 above surface -- oh, they go $\frac{2}{5}$, $\begin{array}{r} 100 \\ -\ \frac{2}{5} \\ \hline 99\frac{3}{5} \end{array}$."

Case U

P: "$\frac{2}{5}$ under water, $\frac{3}{5}$ on surface. Does that mean how many hours?"

E: "Yes, it means hours."

P: "$\frac{3}{5} \times \frac{100}{1} = 60$ miles on surface. $\frac{2}{5} \times \frac{100}{1} = 40$ miles under water. $8\overset{\displaystyle 5}{\overline{)40}}$ under water. Got to divide 60. $15\overset{\displaystyle 4}{\overline{)60}}$ hours. $\begin{array}{r} 5 \\ 4 \\ \hline 9 \end{array}$ hours -- I got it!" (Pupil was sure of his answer and did it in a most straightforward manner.)

Case X

P: "Take $\frac{2}{\cancel{5}}$ of $\overset{20}{1\cancel{0}\cancel{0}} = 40$ = miles under water it would go. Then take 8 into 40, $8\overset{\displaystyle 5}{\overline{)40}}$ hours. I am going to subtract 40 from 100. $\begin{array}{r} 100 \\ 40 \\ \hline 60 \end{array}$ top of water, divide 60 by 15, $15\overset{\displaystyle 4}{\overline{)60}}$, and add $\begin{array}{r} 4 \\ 5 \\ \hline 9 \end{array}$ hours to cross channel."

E: "Why divide 8 into 40?"

P: "To see how many hours."

8-C. If $3\frac{1}{2}$ tons of coal cost \$21, what will $5\frac{1}{2}$ tons cost? (47.8%)

Case J

P: "I think I'd multiply $5\frac{1}{2}$ tons x \$21."

E: "Will you show me how you got that answer?"

P: "
$$\begin{array}{r} 5\frac{1}{2} \\ \underline{21} \\ 2\overline{)21} \\ 10\frac{1}{2} \\ 5 \\ \underline{10} \\ \$115\frac{1}{2} \end{array}$$
It couldn't be $\$115\frac{1}{2}$ so it is $\$1.15\frac{1}{2}$."

E: "Where did you get the decimal point?"

P: "Supposed to count two for a dollar."

Case O

P: "I am going to multiply $3\frac{1}{2} \times 5\frac{1}{2} = 16$."

E: "Why are you going to multiply?"

P: "Well, what would be the cost of 5 tons."

E: "How did you get 16?"

P: "Five times 3 = 15 and $\frac{1}{2}$ and $\frac{1}{2}$ = 1 and
$$\begin{array}{r} 15 \\ \underline{1} \\ 16 \end{array}$$
."

E: "What are you doing now?"

P: "I am adding
$$\begin{array}{r} 21 \\ \underline{16} \\ \$37.00 \end{array}$$
."

E: "Why did you add 21 and 16?"

P: "To see what it would cost."

Case R

P: "Wouldn't you find cost of 1 ton? Oh, wait, let's see. Divide $3\frac{1}{2}$ by 21 = $\frac{7}{2} \times \frac{1}{21} = \frac{1}{6}$. Oh, don't even know what I am doing. I am going to try to multiply that out. $\frac{1}{6} \times 5\frac{1}{2} =$. Now, look, if I take 1 off here, ($5\frac{1}{2}$) and put it on here, ($3\frac{1}{2}$) that would be 4 added to 7 --"

E: "Where did you get 7?"

P: "From 3 into 21. 21 and 7 makes 28. Then I took another from this $5\frac{1}{2}$ and added on to $3\frac{1}{2}$, then added $\begin{array}{r} 28 \\ \underline{7} \\ 35. \end{array}$

$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2} = 1\frac{1}{2}$, you can't do money that way so it makes this $\begin{array}{r} 28 \\ \underline{7} \\ 3\cancel{5}6 \end{array}$ -- I'd like to know what one ton would cost. If I could only get that. $\frac{1}{6} \times$ -- $\frac{7}{2} \div \frac{21}{1} = \frac{1}{6}$. (Pupil knew \$6 would be the cost of one ton.) $\frac{1}{6} \times 5\frac{1}{2} = \frac{11}{12}$, is what I got."

Case S

P: "I'd say divide $3\frac{1}{2} \times 21$. $3\frac{1}{2} \times \frac{1}{21} = \frac{7}{2} \times \frac{1}{21} = \frac{1}{6}$. Oh, dear! A crazy answer -- I got $\frac{1}{6}$."

E: "Is that answer reasonable."

P: "I didn't get that one right. (Pupil returned to this problem and worked it again.) I multiplied $5\frac{1}{2} \times 21$, $5\frac{1}{2} \times 21 = \frac{11}{2} \times \frac{21}{1} = \frac{231}{2}$, $5\frac{1}{2} \times 21 = \$1,110$. It don't sound reasonable, though."

Case T

P: "Let's see, I'd -- 21 into $3\frac{1}{2}$ -- $\frac{21}{1} \div 3\frac{1}{2} = \frac{21}{1} \times 3\frac{1}{2} =$

$\frac{\cancel{21}^{\,3}}{1} \times \frac{2}{\cancel{7}} = \frac{1}{6}$. I multiplied 6 into 5, $6 \times 5\frac{1}{2} = \$33.00$."

Case V.

P: "I am multiplying $3\frac{1}{2} \times 2000$."

E: "Why?"

P: "Because 2000 pounds in one ton."

E: "Read your problem out loud for me, please." (Pupil read problem.)

P: "
$$\begin{array}{r} 2000 \\ \underline{3\frac{1}{2}} \\ 2\overline{)2000} \\ 1000 \\ \underline{6000} \\ 61,000 \end{array}$$
lbs."

E: "Is that your answer, 61,000?"

P: "I have to do something with this $5\frac{1}{2}$.
$$\begin{array}{r} 21 \\ \underline{5\frac{1}{2}} \\ 2\overline{)21} \\ 10\frac{1}{2} \\ \underline{105} \\ 115\frac{1}{2} \end{array}$$
or \$116.00 for $5\frac{1}{2}$ tons of coal."

Case W

P: "Six 3's 18, and 3 more, 21. \$6.00 a ton, \$3.00 a half ton, 11 half tons in $5\frac{1}{2}$. It would be \$33.00 or wouldn't it?

E: "Where did you get 6?"

P: "Well, 6 and 6 and 6, 18, and 3 are 21.
$$\begin{array}{r} 6 \\ 6 \\ \underline{6} \\ 18 \end{array} \qquad \begin{array}{r} 18 \\ \underline{3} \\ \$21 \end{array}$$
for $3\frac{1}{2}$ tons.
$$\begin{array}{r} 6 \\ 6 \\ 6 \\ 6 \\ 6 \\ \underline{3} \\ \$33.00 \end{array}$$
"

E: "Where did you get your 3?"

P: "That's your half ton."

Case X

P: "You'd find cost of 1 ton. $3\frac{1}{2} \div \frac{21}{1} = \frac{7}{2} \times \frac{1}{21} = \frac{1}{6}$, no, \$6.00. $6 \times 5\frac{1}{2} =$ \$33.00 for $5\frac{1}{2}$ tons."

6-C. A dealer bought some mules for $800. He sold them for $1,000, making $40 on each mule. How many mules were there? (60.8%)

Case J

P: "If you make $1000 on all the mules, and $40 on one you would take 40 into 1000. $40\overline{)1000}^{\,25}$."

E: "Did you make $1000 on all the mules?"

P: "He sold them for $1,000 and he got $40 for each mule. (Reread problem.) I think I should have subtracted something."

Case Q

P: "

```
 800      1,000
  40        .40
 ---      -----
 000        960   That's what I get. I checked it."
3200         40
 ---      -----
          1,000
```

E: "Why did you put a decimal point in front of the 40?"

P: "Because I was subtracting."

Case T

P: "I'd subtract 800 from 1000, $\begin{array}{r}1000\\ \underline{800}\\ 200\end{array}$, then I'd divide $40\overline{)200}^{\,5}$. He had 5 mules."

Case U

P: "800 divided by 40 -- 40 each mule -- how many mules-- $40\overline{)800}^{\,\$20}$ each mule, that isn't right, either. (Reread problem.) He made $200 -- $40 -- $40\overline{)200.00}^{\,5.00}$, 500 mules --that isn't right, either."

E: "Why didn't you think that right?"

P: "I don't know, have to divide to find out. 500 couldn't be right. (Pupil called 5.00, 500.)

$\begin{array}{r} 5. \\ 40\overline{)200} \end{array}$ That's right, 5 mules, that's it, 5!"

Case V

P: "You'd divide 40 into 1000. $\begin{array}{r} 25 \\ 4\text{Ø}\overline{)100\text{Ø}} \\ \underline{8} \\ 20 \end{array}$ mules."

E: "Why did you divide 40 into 1000?"

P: "I don't know. If you divide 40 into 800, why you wouldn't make but 2 mules."

Case Z

P: "You would take 40 into 200. $\begin{array}{r} 5. \\ 40\overline{)200} \end{array}$ Five mules, because 40 into 200, 5 times. (Pupil had trouble finding quotient.) Five mules."

4-C. If you buy two packages of paper at 7 cents each and a notebook for 65 cents, how much change should you get from a two-dollar bill? (65.2%)

Case K

P: "I don't know how."

E: "Do you want to try it?"

P: "I'll try it but I don't think I can get it. $\begin{array}{r} 65 \\ \underline{7} \\ 455 \end{array}$, then you subtract that from \$2.00, $\begin{array}{r} \$4.50 \\ \underline{2.00} \\ \$2.50 \end{array}$. That doesn't sound reasonable."

Case G

P: "Would you multiply? $\begin{array}{r} 7 \\ \underline{2} \\ 14 \end{array}$ $\begin{array}{r} 14 \\ \underline{65} \\ 79 \end{array}$ $\begin{array}{r} 2.00 \\ \underline{.79} \\ 1 \end{array}$ -- I'd get \$1.21."

E: "Will you show me your work?"

P: "I did it in my head -- I don't see how I did it. I know I subtract. It would be 21¢ to make 79 and \$1.00

left, \$1.21." (Pupil had no idea how to subtract, that is, put it on paper.)

Case U

P: "Two packages,

$$\begin{array}{r} 14¢ \\ \underline{65¢} \\ \$.79 \end{array} \qquad \begin{array}{r} 1.00 \\ \underline{.79} \\ \$\ .21 \end{array}$$

change. That's how much he got."

E: "Is that answer reasonable?"

P: "Yes, 21¢ from 1 dollar."

Case Z

P: "14¢ for paper,

$$\begin{array}{r} 14¢ \\ \underline{65} \\ 79 \end{array} \qquad \begin{array}{r} 2.00 \\ \underline{79} \\ \$1.21 \end{array} \qquad \begin{array}{r} 2.00 \\ \underline{79} \\ 1.21 \end{array}$$

."

2-C. How many hours will it take a truck to go 66 miles at the rate of 6 miles an hour? (78.1%)

Case O

P: "I think you'd divide, wouldn't you?"

E: "Go ahead and work it."

P: "Divide -- 6 will go into 66, $6\overline{)66}^{\,11}$ times."

E: "What is the 11?"

P: "What do you mean? Oh, -- it's eleven hours."

Case V

P: "Oh, goodness! Multiply 66 by 6 -- no! $\begin{array}{r} 66 \\ \underline{6} \\ 396 \end{array}$."

E: "Why did you say 'no'?"

P: "It wouldn't take no 396 hours. I don't know what you mean by that problem." (Pupil returned to this problem later.) "Divide 6 into 66, $6\overline{)66}^{\,11}$."

E: "Why?" (Pupil began to erase work.)

P: "I guess that's wrong."

E: "I didn't say your answer was wrong, I just asked why."

P: "I don't know why." (Pupil left the work as it was.)

Case Y

P: "16 is the answer."

E: "How did you get it?"

P: "10 x 6 = 60
6 x 11 = 61."

E: "Where did you get your 11?"

P: "Well, see, 6 x 11 = 61.
12 x 6 = 62
13 x 6 = 63
14 x 6 = 64
15 x 6 = 65
16 x 6 = 66."

E: "Why did you take 10 x 6 = 60?"

P: "Well, I just thought of that first. 6 into 66 goes 16 times." (Pupil used the right method but couldn't divide or multiply and was incorrectly adding until the desired number, 66, was reached.)

3-C. A regiment marched 40 miles in five days. The first day they marched 9 miles, the second day 6 miles, the third 10 miles, the fourth 8 miles. How many miles did they march the last day? (82.5%)

Case N

P: "Add 9, 6, 10, 8. 8 and 8 are 16 and 1 are 17, (Pupil split combinations) and 6 are 23, and 10 are 33. Seven miles."

$$\begin{array}{r} 40 \\ \underline{33} \\ 7 \end{array}$$

Case P

P: "They marched 8 miles."

E: "Where did you get your 8?"

P: "It says last day." (Pupil was using 4th day.)

E: "Perhaps that isn't what it means."

P: "Oh, yeah. You add $\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \\ 33 \end{array}$ $\begin{array}{r} 40 \\ \underline{-33} \\ 7 \end{array}$ miles last day."

Case Q

(Pupil wrote the numbers $\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \\ 34 \end{array}$, but in adding out loud it was noticed the pupil split the combinations.)

P: "Nine and 6 are 15 and 4 are 19 and 4 are 24 and 10 are 34. $\begin{array}{r} 40 \\ \underline{34} \\ 6 \end{array}$ miles last day."

Case R

P: "You'd add. Let us do all these (pointing to the numbers) $\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \end{array}$ and see if they come out 40. I'd have to try that out, wouldn't I? $\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \\ 53 \end{array}$ $\begin{array}{r} 53 \\ \underline{40} \\ 13 \end{array}$. That isn't right, is it?"

Case T

P: "I add and then subtract. (The pupil wrote the numbers $\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \\ 33 \end{array}$, but in adding did not follow the sequence of his numbers.) 9 and 8 are 17 and 6 are 23 and 10 are 33. Divide $33\overline{)40}^{\,1}$ 1 7/33 miles."

Case V

P: "You'd add all them together and then subtract from 40."

$$\begin{array}{r} 9 \\ 6 \\ 10 \\ \underline{8} \\ 23 \end{array}$$

E: "Add out loud for me, will you?"

P: "I take large numbers first, 9 and 8 are 17 and 6 are 23. $\begin{array}{r} 40 \\ \underline{23} \\ 17 \end{array}$. No (humorously) I have to add 10 more makes 33. Forty minus 33, $\begin{array}{r} 40 \\ \underline{33} \\ 7 \end{array}$ miles."

1-C. If 24 men are divided into squads of 8, how many squads will there be? (91.3%)

Case K

P: "Multiply $\begin{array}{r} 24 \\ \underline{8} \\ 192 \end{array}$ squads."

E: "Why did you multiply?"

P: "Well, because it was the only way you could get it, Mister."

E: "What do you mean?"

P: "That's the only way you could find out how many squads there would be."

Case T

P: "I'd divide 8 into 24 -- let's see, it would go 3 times -- 3 x 8 is 24."

E: "What is that 3?"

P: "Three squads."

11-C. A boy is five years old and his father is 35 years old. If his uncle is 40 years, hold old will his cousin be?

Case F

P: "Subtract father's age from uncle's.

$$\begin{array}{rl} 40 & \text{Uncle's} \\ \underline{35} & \text{Boy's father} \\ 5 & \end{array}$$

Comes out 5. Then you add that 5 to age of boy,

$$\begin{array}{r} 5 \\ \underline{5} \\ 10 \end{array}$$

and comes out 10 years."

E: "Is that reasonable?"

P: "I believe so. I could do it another way. I could subtract boy's age, 5, from father's,

$$\begin{array}{r} 35 \\ \underline{5} \\ 30 \end{array}$$

, and then

$$\begin{array}{rl} 40 & \text{Uncle's age} \\ \underline{30} & \text{difference between boy's and father's age} \\ 10 & \text{years old his cousin would be."} \end{array}$$

Case M

P: "It doesn't tell when his cousin was born. How do they know his cousin had -- I am going to leave that one. If it said his uncle's was 32 when his cousin was born and how old would his cousin be now, it would be easy."

Case O

P: "I'd subtract 40 minus 35 = 5 years. He is 5 years old."

Case S

P: "Mmmmm -- I'd divide $5\overline{)35}^{\,7}$ years old."

E: "Does that sound reasonable?"

P: "Yeah!"

Case T

P: "Oh, his cousin will be 35."

E: "How did you get it?"

P: "Five from 40, $\begin{array}{r} 40 \\ \underline{5} \\ 35 \end{array}$."

Case X

P: "I am going to add 5 and 35 = 40."

E: "Where did you get 5?"

P: "Well, boy's 5. I think I'll subtract it. $\begin{array}{r} 35 \\ \underline{5} \\ 30 \end{array}$."

E: "What is that 30?"

P: "$\begin{array}{r} 35 \\ \underline{5} \\ 30 \end{array}$ father's age when boy was born. $\begin{array}{r} 40 \\ \underline{30} \\ 10 \end{array}$. Cousin 10 years old."

Case Z

P: "I don't know how to tell how old his cousin would be. It would depend on how old the uncle would be when he had his boy, whether uncle married and how old when he had his boy."

12-C. If a fencing costs 80 cents a foot, how much will it cost to put a fence around a garden 40 feet long?

Case F

P: "$\begin{array}{r} \$\ .80 \\ \underline{40} \\ \$32.00 \end{array}$ It cost \$32.00 to put it around the garden."

E: "Does your answer sound reasonable?"

P: "Yes."

Case M

P: "Doesn't tell how wide it is."

E: "Did you need to know that?"

P: "Yes, it says around the garden."

Case R

P: "How wide is the garden? I guess there isn't any width to it. That's easy -- 80

$$\begin{array}{r} 80 \\ \underline{40} \\ 00 \\ \underline{320} \\ \$32.00." \end{array}$$

Case U

P: "

$$\begin{array}{r} 40 \\ \underline{80¢} \\ \$32.00 \end{array}$$

It would cost him $32.00. I got that one! You multiply -- it's $32.00."

Case V

P: "That would be 40¢."

E: "How did you get it?"

P: "I subtracted 40 from 80.

$$\begin{array}{r} 80 \\ \underline{40} \\ \$.40." \end{array}$$

Case Z

P: "It would depend how wide it was. You can't find it without."

CHAPTER V

THE SUMMARY AND CONCLUSIONS

1. The Problem and Plan of This Study

The purpose of this research was to investigate how pupils solve verbal problems in arithmetic. The investigation involved several related problems, such as; to what extent was the pupils' method of solution influenced by irrelevant data, details, cues, and quantities used in a problem? Data relative to children's preference for certain problems were analyzed and sex differences were studied.

The experimenter employed three methods to obtain data on his problem: (1) Studied the nature of pupils' responses to changes in the statement of a problem by means of a statistical analysis of a written test; (2) analyzed the pupils' written test for further evidence as to the procedure followed by pupils in solving problems; (3) interviewed certain pupils, that is, gave them an oral test to get additional evidence as to the extent of their critical thinking.

The experimental written test for this study was formulated by the experimenter in companion problems. The companion problems were exactly alike in difficulty of computation and method of solution except for one factor. Each child worked the paired problems. The hypothesis of the experimenter was that if a significant difference were found it could be attributed to the experimental factor. The written test was given in two parts, Tests A and B, making a total of forty problems. The first ten problems in the oral test, Test C, were adapted or taken directly from the Army Group Examination Alpha. An additional two problems were included in the oral test that were impossible of a correct solution. The oral test took, on the average, a little over an hour and fifteen minutes to give.

Considerable research was done to make the written test, Tests A and B, valid and reliable. The experimenter studied other reasoning tests for this level, textbooks were consulted, and related studies were of particular value. Many of the problems were selected or adapted from other tests. The coefficient of reliability for the written test is .935 $\pm$.005, which is evidence that the test is a reliable instrument for measuring the abilities in question. This coefficient does not insure the test is valid, but it does indicate that the possibilities exist for it to be valid.

The experimental group was comprised of 518 sixth grade pupils in the following cities in Kansas: Pratt, Haven, Russell, Norton, Ellis, Kinsley, Stockton, Hays, and Rural Districts 12 and 59 in Ellis County. Pupils of fifteen different teachers were represented. The oral test was given to twenty-three pupils in the four sixth grade classes in Hays and in District 59. Each of these five classes was taught by a different teacher. Those who took the oral test had first taken Tests A and B, the two parts of the written test.

Tests A and B were administered on two consecutive days in the last two weeks of March, 1942. The oral test was given in the following week. A scheme was devised so that half the experimental group took Test A the first day and half took Test B the first day. This was done to eliminate as much as possible the effect of practice and related problems. The written test was administered by the experimenter or under the direction of the administrative head of the school. In every instance, the written test was administered by one experienced in testing. The experimenter gave all the oral tests.

2. The Specific Conclusions

The findings in this study appear to support the theses that:

(1) Pupils do not discriminate between relevant and irrelevant data. This is suggested in Table I. They do not select pertinent material, but tend to compute with

whatever quantities they find in a problem with little regard for the purpose of the quantities.

(2) Pupils, as the findings in Table II indicate, are not greatly effected by details. They are about as successful with problems without details or abstract problems as they are with problems written with details or in concrete form.

(3) Pupils make unthinking responses when they come upon familiar cues in a problem, such as average, how many times, perimeter, etc. They "apparently reason" correctly when a certain cue in a division problem indicates that they should divide, but when presented with a problem that requires the "same reasoning" they may multiply in the absence of the familiar cue. The differences found in Table III and analysis of pupils' work support this conclusion.

(4) Pupils appear to be unduly influenced by the quantities employed in a problem. This is shown in Tables IV and V. They do not seem to think in terms of even the simplest fraction, but rather appear to have pre-conceived notions as to how a problem should be worked before it is carefully read. There is apparently little transfer of knowledge in solving problems in which integers are used and in solving nearly identical problems in which simple fractions are employed.

(5) Pupils do not necessarily like problems they can

work, as shown by the correlations on page 34, but inability to work a problem does seem to be a fair indication that they will not like it.

(6) Pupils of either sex can not be expected to excel the other. Table VII reveals that there is no significant difference in the ability of boys and girls to solve verbal problems in arithmetic.

In general, the experimenter believes it is a justifiable conclusion that a large per cent of sixth grade pupils do not follow a rational procedure in their attempt to solve verbal problems in arithmetic, even though their activity is usually purposeful. There is little evidence that they appraise the total situation before attempting to solve problems but rather make stereotyped responses to certain phrases and quantities used in a problem.

3. The Practical Conclusions

Throughout this study the evidence has suggested that pupils tend to make unthinking reactions to the data found in a problem. The role of habit appears to be overemphasized in the teaching of arithmetic. Better ways of acquiring mastery of reasoning problems are to be found than having pupils make habitual responses to problems that require thinking. A habit at its best can only contribute to thinking, it can not replace thinking. The kind of thinking described on page 38 is difficult to acquire but it is well worth acquiring, and in the end it may be more economical for the pupil to do so. Naturally the problem must be on the level of the pupil's ability.

The experimenter wishes to point out that he does not contend that the arithmetic curriculum should be so constructed that difficult elements in problem solving should be eliminated. The question is not which type of problems will be the easier to solve or to grade, but which will better prepare the pupil for critical thinking in the practical situations he will meet in life. It does not follow that practical problems are always interesting, or always contain only relevant material. The fact that it is "a problem" precludes that it can be in a form that a child can solve without thinking.

An analysis of the pupil's mental processes that lie back of their answers often reveal that their errors are due to faulty habits of thinking and not because they do not have the ability to think. Improvement can be, and must be, make in teaching children to think. The type of instruction and the kind of verbal problem that will facilitate and develop ability to think is a problem that seems to lie deeper than the one here investigated.

APPENDIX

TEST A

1. At a bargain sale the clerk sold shirts for $1.84. What was the bargain price for 7 shirts? (86.6%)

2. A group of 5 hunters paid the sum of $7.25 for rent of a lake. Find the average cost for each hunter. (85.3%)

3. A pennant was cut so that its base was 3½ ft., the top side was 8¼ ft., and the lower side was 8½ ft. What was the perimeter of the pennant? (62.5%)

4. A poor girl must pay $23.40 for a glass door she broke when a child pushed her against it. There are 45 children in our class and we decided to share equally the cost of the door. What must each one pay? (76.0%)

5. Mr. Miller left $90 of his money to his only daughter and $400 of his money to be divided equally among his five sons. How much did each son receive? (82.0%)

6. A pole 6 ft. long is how many times as long as a stick 2/3 of a ft. long? (19.1%)

7. Some toweling costing 72¢ per yd. is cut into lengths 3/4 yd. each. Find the cost of the material in each towel. (42.4%)

8. A group of 17 persons agreed to give the sum of $18.50 to a very poor family. How much will each person's share be, if they all agree to give the same amount? (68.1%)

9. The list price is $4.50; the discound is $1.85. What is the net price? (64.0%)

10. A gruit grower raising pears for market finds that he grew 84 bu. on each acre. What was the total bu. raised on 14 acres? (73.3%)

11. A man started on a trip with 8 gallons of gasoline in his car. At the end of the day he had 4 gallons left. He had bought 6 gallons on the way and had traveled 150 miles. How many miles did he get to a gallon of gasoline that day? (42.8%)

12. If 1½ chocolate cakes are enough for a picnic table, how many tables will 18 cakes supply? (24.5%)

13. In drilling his oats, a farmer plans to use 3/4 bu. of seed oats per acre. How many bushels will it take to plant 24 acres? (67.3%)

14. Because of the war I can only buy 12 oz. of sugar for each person per week. There are 6 persons in our family. How many oz. of sugar can I buy this week? (83.5%)

15. A boy had 210 marbles. He lost 1/3 of them. Of the ones he lost 20 were new. How many had he left? (20.2%)

16. Tom paid $17.10 for 18 yards of tent canvas. Find the cost of one yard. (62.5%)

17. The girls can make a doll house in 48 hours. They are working on it 2 hours a day. How many days will it take to finish the doll house? (67.3%)

18. I used 132 gallons of gasoline in a car that usually makes 14.5 miles on one gallon. How long a trip did I take? (29.7%)

19. Mr. Smith has a 45-acre farm. If 25 acres are meadow, what part of his farm is meadow? (28.4%)

20. Mr. Jones hired boys to dig a cellar 11 yds. long, 8 yds. wide, and 3 yds. deep. How many cubic yds. of dirt were taken out in digging the cellar? (19.6%)

Which two of the 20 problems on this test did you like best?
Which two of the 20 problems on this test did you not like?

TEST B

1. The store sold chickens for $1.68. Find the price of 8 chickens. (81.6%)

2. A group of 7 fishermen paid the sum of $8.75 for a boat. Find the cost per fisherman, if they are all to pay the same amount. (81.8%)

3. A pennant was cut so that its base was $3\frac{1}{4}$ ft., the top side was $8\frac{1}{2}$ ft., the lower side was $8\frac{1}{4}$ ft., and its altitude $3\frac{1}{2}$ ft. What is the perimeter of the pennant? (6.5%)

4. The bill is $24.30. If 45 persons share equally the cost of the bill what must each one pay? (74.1%)

5. A farmer left 260 bushels of wheat to be divided equally among his four sons. How much did each son receive? (86.1%)

6. A rope 12 yds. long could be cut into how many pieces that are 3/4 of a yard long? (29.1%)

7. If a roll of ribbon priced at 85¢ per yard is made into hair ribbons of 3/5 yds. apiece, what will be the value of each hair ribbon? (45.1%)

8. A group of 19 adults agreed to give the sum of $20.00 for Red Cross. Find the average amount for each person's share. (66.2%)

9. The list price of caps is $3.30. During a sale, Walter bought a cap on which a discound of $1.45 was given. What is the net price Walter paid? (73.9%)

10. A fruit grower raising apples for market finds the average yield of an acre to be 91 bu. Find the yield for 13 acres. (61.1%)

11. Last summer Agnes Purdy, her brother Archie, and their parents took a trip in their Ford. Archie measured the gasoline when they started. "We have eight gallons", he told his father. At the end of the day he found 4 gallons of gasoline in the tank. They had bought 6 gallons at a station on the way and had traveled 160 miles. Agnes told her Mother that they had made____miles to a gallon of gasoline that day. (40.3%)

12. If 2 cherry pies are enough for a picnic table, how many pies will it take to supply 12 tables? (83.5%)

13. In drilling his wheat, a farmer plans to use 3/4 bu. of seed wheat per acre. How many acres will 24 bushels plant? (20.6%)

14. I can buy 13 lbs. a week per person. There are 5 persons. How many lbs. can I buy this week? (84.5%)

15. A girl had 90 jacks. She lost 1/3 of them. How many had she left? (19.8%)

16. Sally paid $16.15 for 19 yards of curtain material. How much did she pay for one yard? (60.8%)

17. The boys can build a boat in 36 hours. They are working on it 3/4 of an hour a day. How many days will it take to finish the boat? (19.3%)

18. If a car can run 15.5 miles on one gallon of gasoline, how far will it run on 124 gallons? (35.3%)

19. What is the ratio of the speed of a steamship which travels 25 miles an hour and the speed of a railroad train which travels 40 miles an hour? (8.3%)

20. A crew of men working for 8 hours with a steam shovel, dug a basement 9 yds. long, 10 yds. wide, and 3 yds. deep. Mr. Thomas paid them 40¢ a cubic yd. for this work. How many cubic yds. of dirt were taken out in digging the hole? (13.3%)

Which two of the 20 problems on this test did you like best?

Which two of the 20 problems on this test did you not like?

TEST C

1. If 24 men are divided into squads of 8, how many squads will there be? (91.3%)

2. How many hours will it take a truck to go 66 miles at the rate of 6 miles an hour? (78.1%)

3. A regiment marched 40 miles in five days. The first day they marched 9 miles, the second day 6 miles, the third 10 miles, and the fourth 8 miles. How many miles did they march the last day? (82.5%)

4. If you buy two packages of paper at 7 cents each and a notebook for 65 cents, how much change should you get from a two-dollar bill? (65.2%)

5. If it takes 6 men 3 days to dig a 180-foot drain, how many men are needed to dig it in half a day? (13.0%)

6. A dealer bought some mules for $800. He sold them for $1,000, making $40 on each mule. How many mules were there? (60.8%)

7. A rectangular bin holds 400 cubic feet of lime. If the bin is 10 feet long and 5 feet wide, how deep is it? (17.3%)

8. If $3\frac{1}{2}$ tons of coal cost $21, what will $5\frac{1}{2}$ tons cost? (47.8%)

9. A ship has provisions to last her crew of 500 men 6 months. How long would it last 1,200 men? (4.3%)

10. If a submarine makes 8 miles an hour under water and 15 miles on the surface, how long will it take to cross a 100-mile channel, if it has to go two-fifths of the way under water? (21.7%)

11. A boy is five years old and his father is 35 years old. If his uncle is 40 years, how old will his cousin be?

12. If a fencing costs 80 cents a foot, how much will it cost to put a fence around a garden 40 feet long?

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