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A Factor Analysis of Verbal Ability

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A FACTOR ANALYSIS OF VERBAL ABILITY

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TABLE OF CONTENTS

List of Tables

INTRODUCTION

For many years the practical value of tests of intelligence, aptitude and other qualities has been demonstrated. Their practical value is shown by their growing usefulness in: vocational guidance, educational research, remedial education, educational administration, institutional commitments, industrial efficiency, and social reform.

As the "testing movement" has grown, a multitude of tests has been devised and is now available for the testing of human abilities. Each test is unlike other tests although there are vague similarities running through groups of tests. As it is impossible to give all the tests devised, necessity leads one to search for a few tests that will describe the individual as accurately as, but with greater simplicity than, the thousand and one tests now available. It is with this idea in mind that several modern psychologists have attempted to study the organization of mental ability from various theoretical points of view with various

mathematical methods. A brief review of some of these attempts will provide the context for the present investigation.

Two of the important attempts to understand intellectual abilities are found in the theories and methods of Spearman (12) and Thurstone (16). These two are representative of the status of research in this field today. Spearman, the exponent of the two-factor theory, believes that all correlations between mental tests are due to one factor present in all the tests. This factor, common to all tests, is labelled "g", indicating the general similarity running through each test. The part of the tests that is not included in the correlation is believed to be specific in nature and unique for that test in that it is not to be found elsewhere, except in tests of a similar nature. The description of two factors accounts for the title, the Two-Factor Theory. Spearman, to measure the amount of "g" in each test, devised a special mathematical procedure--the tetrad method--for analyzing a correlation table involving many tests. According to his theory it would be possible to measure the amount of an individual's "g", which would reveal nearly everything about some of

his abilities and something about nearly all of them. The same can be done for his specific factors, exhausting the information supplied by the tests.

Holzinger's bifactor method (6) came later as a natural improvement or extension of the two-factor theory of Spearman's to include group factors as well as the "g" factor. In many ways it can be looked upon as a three-factor theory as some tests may have something in common which is not the "g" factor nor a specific factor. The mathematics is somewhat more sophisticated than Spearman's tetrad formula.

Sharply contrasted with Spearman's Two-Factor Theory is Thurstone's Multiple-Factor Theory. He is of the opinion that analysis of a battery of tests will disclose several different factors, depending upon the nature of the tests. He sees no reason for assuming a general factor. Thurstone has devised several methods for factoring a correlation matrix into a small number of factors. One of these, the centroid method, is used in the present study and will be described later. Although there are other multiple-factor methods, such as Kelley's (7), Thurstone's is the most widely discussed today.

The factor analysts have attacked their problems with the hope that they might obtain a few single tests or a weighted battery of tests that approximate the factors found. With these primary tests they could estimate the amount of each factor possessed by any man with a view to predicting and guiding his vocational and educational future.

There is another line of psychological research, apparently independent of the above-mentioned statistical studies, which might also throw an interesting light on mental activity. Experiments on the psychology of thinking or problem-solving have been directed toward discovery of the various processes involved in complex mental activity, and the reasons for success and failure on difficult intellectual tasks. Individual differences have not been studied so thoroughly as in the statistical approach.

There is a possibility that the statistical studies outlined above have overlooked a valuable source of information, that is, existing experimental information about different processes included in mental ability. It is this possibility that prompted the present study.

The statistical investigations outlined first have resulted in factors which are usually named from different materials, or different kinds of problems, i.e., verbal and numerical. It would also be instructive to search for various processes or functions of thought. We may find clues to guide the search for significant processes in the speculations and experiments of the philosophers, logicians and psychologists.

Familiarity with the experimental literature on the higher mental processes, such as Woodworth has summarized in the last two chapters of his recent work (18), suggests that there may be two fundamental processes involved in solving all problems: (1) calling up various acts or responses, and (2) selecting those responses which meet the requirements of the problem. Comparative psychology teaches that in the first process, calling up responses, the higher, more intelligent species show more variability. They have a larger repertoire of responses. Munn states, for example, that "stereotypy decreases and general plasticity of response increases as the higher forms are approached" (8, p. 100). Clinical psychology

shows that the bright use more different methods of performing than the dull. Experiments on human adults give similar results and suggest that individual differences in this process may be an important factor in solving problems. Let us call this hypothetical factor F, (for flow of responses).

But problems are not consistently solved by aimless activity alone. The problem-solving experiments, both animal and human, show restriction of the responses, under the influence of some sort of directive tendency, to the more likely places, objects or ideas. Complete solution, including transfer, implies recognition of the fit of certain responses to the requirements of the problem. Let us assume that individual differences in this process may be another important factor in solving problems. Let us call this hypothetical factor S, (for selection of responses).

Animals, in solving problems, display a variety of movements which seem to have some degree of plausibility. They seldom lose sight of their goal so far as to thrash about with movements that bear no relationship to the objective situation. They appear to use a trial-and-error method of behavior

which is directed toward solution but very few records show where such behavior was guided from the beginning by perception of the exact movements necessary for solution. Their actions may be considered exploratory. They try movement after movement and gradually the unsuccessful attempts are eliminated.

The animal's activity, although varied and apparently not confined always to a logical solution, indicates exploration and testing of various responses. Thus, we find the two processes--F and S--present in the problem-solving activity of animals. The varied movements and activities reveal the flow of responses directed toward solution. Selection of responses appears in the elimination of unsuccessful activity and a gradual narrowing of the field of activity as the proper movements are selected. Concommitantly the activity seems to have less of a trial-and-error character. Woodworth (18, pp. 749-752) commenting on different works said:

> Thorndike found all but the hardest problems to be solved by a rapid, often apparently instantaneous, abandonment of the unsuccessful movements and a selection of the appropriate one . . . Thorndike had spoken of "acts" or movements, which became associated with the situation as a whole rather than of objects with which the animal dealt. Thus there grew up a picture of trial and error as consisting of varied motor responses to a

total situation, these movements which resulted well gradually getting advantage over the rest.

The human being shows a greater variety of responses, although restricted, than does the animal. His responses or movements are largely a result of past experiences as he perceives the relationship of the present task with problems of a similar nature which has confronted him in the past. The human being apparently has a great deal of insight into problems as a sudden solution of many tasks indicates "seeing through the situation". The fact that a human being does have insight is due to his knowledge and experience. The human calls up fewer responses or impulses as if following a plan carefully directed toward solution. The problem provides the stimulus and the responses selected are determined by the relationship of the problem to past experience and knowledge. Selz's anticipatory "schema" in thinking, as stated by Woodworth (18, pp. 799, 800), explains this more clearly.

> The schema is a plan of operation, a method of solution. Sometimes the operation is very direct and simple and amounts to Spearman's "eduction of a correlate." The task prescribes a certain relation, the stimulus provides one "fundament," and the blank form needs only the other fundament to be complete.

We do not know, of course, whether flow of one kind of responses, e.g. verbal, is related to flow of another kind, e.g. numerical. Hence, the investigation has been confined to verbal tasks in order to limit the field and guide the procedure. Furthermore, verbal factors have appeared to be most significant in analyzing intellectual ability. Anastasi (1, p. 296) has the following to say about certain factors involved in intelligence:

> Most paper-and-pencil tests . . . measure chiefly verbal ability and, to a slighter extent, numerical ability. Since the latter type of test (paper-and-pencil) is by far the most frequently employed, the term "intelligence" has come to be used almost synonymously with verbal ability.

Since the verbal factor has consistently appeared as a factor of considerable importance in statistical analyses of intellectual ability (see 2, 3, 6 and 14), any information regarding the processes involved in solving verbal problems would, therefore, advance our knowledge of intellectual ability. The central hypothesis is that factor analysis of a number of verbal tests will disclose Factor F, flow of responses, and Factor S, selection of responses, in significant amounts.

It would be difficult to set up an experiment to isolate either of these processes. In operation each is complicated by the other. Individual differences in the two processes may be studied, however, by factor analysis. Factor analysis is a statistical tool used to isolate the fundamental abilities, causes, or variables of mental life. It reveals the dimensions or categories of mental ability. Thurstone (16, p. 189) states:

> Factor analysis is not restricted by assumptions regarding the nature of the factors, whether they be physiological or social, elemental or complex, correlated or uncorrelated. It assumes that a variety of phenomena within the domain are related and that they are determined, at least in part, by a relatively small number of functional unities, or factors.

The names for these factors depend largely upon one's preference and manner of speech or on how much one knows about the domain being investigated. It seemed likely that, if a suitable battery of verbal tests could be given, factor analysis of the scores would yield useful information as to the presence and importance of the processes which have been emphasized in the introduction.

The Tests

The selection of tests for factor analysis is an important matter. In order to measure certain factors, tests have to be chosen that are presumed to have the factors in question in varying quantities. For the present purpose tests had to be chosen that covered a wide range of verbal ability. There should be some tests which could be presumed to include F and S in varying amounts. Thurstone (16) advocates the use of simple tests so that the resulting statistical factors--whatever they might turn out to be--can be interpreted without ambiguity. In order to do this five simple verbal tests were constructed. Ten tests were used in all, five of which were standard tests and five were constructed ad hoc. A description of the tests and a tentative analysis of their content follows. More complete description of the tests in statistical terms will be possible after the factor analysis is complete. See Appendix A for copies of the five original constructions--Tests 1, 2, 3, 4 and 6.

1. Flow. This test was intended to measure F, or flow of responses, in as pure a form as possible. It was suggested by a test on the ten-year level of the old Stanford-Binet (13). The directions were to write as many words as possible in two minutes.

2. Multiple completion. This test was similar to Flow but required some selection or fitting of the responses. Sample:

The horse pulled back____ ____ ____ ____ ____

Each blank was to be completed by a word which, if used alone, would complete the train of thought of the sentence or phrase. Twelve minutes.

3. Paired Opposites. The instructions of this test were to name as many words and their opposites as possible in four minutes. This test was expected to depend largely upon flow of responses. Some degree of restriction is evident, however.

4. Word Cross-outs. Instructions were given to cross out irrelevant words that had been inserted into the text of an article taken from a popular magazine. The words inserted were obviously incorrect in the capacity in which they were used. Nothing was left out of the original theme. For example:

Midget auto racing has developed servile a handicapping system based on the . . .

Servile is to be crossed out. The content of this test could not be estimated in advance. It was included for the sake of variety and motivation. Three minutes.

5. Same and Opposite. This test was taken from the 1931 edition of the A. C. E. test (17). There are four words on a line, two of which are either the same or opposite in meaning. The subjects were to identify the two words. Five minutes. Sample:

1. obtuse, 2. aromatic, 3. acute, 4. occult.

6. Verbal Relations. Two types of verbal problem were used in this test. Samples:

(a) Plank_____meals
(b) Irksome is to wearisome as engrossed is to _____1. enhanced, 2. occupied, 3. apathy, 4. dilapidated.

The problem in (a) was to find a word similar in meaning to both plank and meals, e.g. board. This type of item was suggested by a recent report by Shipley (11). The (b) items were of the usual analogy type. Fifteen minutes.

7. Vocabulary. This test needs but little explanation as it is the vocabulary section of the Nelson-Denny Reading Test (8). It has a statement and five words to select from in answering.

To feign is to 1. fret, 2. faint, 3. molest, 4. pretend, 5. portend.

Ten minutes.

8. Restricted Completion. This test was the completion test of the 1931 A. C. E. examination (17). Sample:

A dead body, especially a human one, is sometimes called a (7)_____.

The number in parenthesis indicates the number of letters in the most appropriate word for that space. This test is restricted in two ways: meaning and length. In test 2, Multiple Completion, there is much less restriction or fitting as the items are more simple and do not require a response limited in length of letters. Eight minutes.

Factor analysis of these eight tests could be expected to disclose some information about individual differences in verbal ability. If a small number of factors should be found, it would be interesting to investigate their involvement in a practically important verbal skill, such as

reading. Therefore, a reading test was included. As the usual intelligence test is usually made up of many verbal problems, an intelligence test was included to see what effect verbal factors would have upon intelligence-test scores. Intelligence-test scores are usually closely related to verbal proficiency.

9. Intelligence. The well-known Henmon-Nelson Test of Mental Ability, Form A, for college students was used (5). Thirty minutes.

10. Reading. The Comprehension section of the Nelson-Denny Reading test, Form A, was used (9). The test was given in accordance with the directions that accompanied the tests. The subjects read paragraphs and answer multiple-choice questions about them. Twenty minutes.

All tests were given as time tests. The times, with the exception of the standard tests, were selected after a little preliminary experimentation.

Subjects

The tests were given during three class hours to 120 summer-school students at Fort Hays Kansas State College. That is, it took three class hours to administer the complete battery of ten tests to

one class. Five different classes were used--two in Psychology, two in English, and one in Speech. The group was somewhat heterogeneous in respect to age since some of the subjects were teachers who had been teaching a few years. The records of 113 subjects were complete and constitute the data for the factor analysis.

Statistical Treatment

This testing program yielded scores for 113 subjects, ten tests per subject, or 1130 test papers. The 1130 papers were scored, and mean scores for each of the ten tests were computed in the usual way. The standard deviations for each of the ten distributions were computed according to the formula, $SD = \sqrt{\frac{\sum d^2}{N}}$. These results are presented in Table I. Table I also shows the reliabilities of each test. With the exception of the Nelson-Denny Reading Test and Vocabulary Tests the reliabilities reported by the author were used. In the case of the Nelson-Denny Reading Test (9), which consists of two parts, Vocabulary and Reading,

TABLE I

MEANS, STANDARD DEVIATIONS, AND RELIABILITIES OF THE TEN VERBAL TESTS

	Test	Mean	Sigma	Reliability
1.	Flow	54	8.3	.603
2.	Multiple Completion	137	34.4	.885
3.	Paired Opposites	21	4.8	.405
4.	Word Cross-outs	21	7.4	.844
5.	Same and Opposite	11	6.2	.87
6.	Verbal Relations	16	4.3	.785
7.	Vocabulary	43	15.2	.842
8.	Restricted Completion	18	6.5	.81
9.	Intelligence	44	10.6	.89
10.	Reading	20	6.3	.842

the reliability of the total score only is given. The assumption was made that both parts were of approximately equal reliability, and the Spearman-Brown "prophecy" formula, $r_{(predicted)} = \frac{2r(obtained)}{1 + r(obtained)}$, was used to estimate the reliability of each part. For the newly constructed tests reliability was computed by the split-half method with the aid of the Spearman-Brown formula. Since two kinds of items were used in Test 6, items 1, 2, 5, 6, 9, 10 . . . were correlated against items 3, 4, 7, 8, 11, 12 . . . . In tests 1, 2, 3 and 4, where the score was simply the number of correct words, the subjects were stopped at half-time and asked to draw a line. Then all were started again at the same place. First and last halves were correlated to get reliability coefficients. Some of these reliability coefficients are probably not highly accurate but they are accurate enough for present purposes. Test 3 has a very low reliability, .405, and does not contribute much to the present investigation. The time for Test 3 was only 4 minutes.

The scores were next transmuted into sigma scores and the intercorrelations among the ten tests

computed by the formula, $r = \frac{\sum AB}{N}$, where A and B are sigma scores. Raw correlations are below the diagonal and corrected correlations above the diagonal. As expected, all correlations are positive, indicating that all tests are loaded with a similar ability--verbal ability. Some of the correlations in Table II are rather high, .829 between Tests 5 and 7, while some are almost zero, .030 between Tests 1 and 8. These high and low correlations were not particularly surprising and coincided with the idea that the writer had in mind when selecting the battery of tests.

Thurstone's centroid method of factor analysis (14) was used in the present study. Its convenience and utility have been pointed out by Garrett (2) and by Guilford (3) among others. Guilford's outline (4) was followed. The analysis was carried through with raw r's and with corrected r's. Since the variance due to unreliability varied considerably from test to test, it seemed to the writer that the factor loadings resulting from the use of corrected r's could be more clearly interpreted. Roff (9) has shown that both correlation matrices have the same

TABLE II

RAW AND CORRECTED CORRELATIONS FOR TEN VERBAL TESTS
(Corrected r's Above the Diagonal; Raw r's Below)

Test	1	2	3	4	5	6	7	8	9	10
1		.415	.573	.310	.297	.275	.181	.030	.135	.185
2	.303		.299	.211	.249	.193	.203	.123	.308	.403
3	.283	.176		.387	.517	.567	.411	.469	.153	.238
4	.221	.179	.226		.525	.650	.577	.452	.750	.695
5	.215	.215	.307	.450		.811	.829	.659	.730	.686
6	.189	.158	.320	.529	.670		.688	.754	.821	.642
7	.129	.171	.240	.486	.710	.559		.548	.665	.717
8	.021	.102	.269	.374	.553	.601	.453		.562	.505
9	.099	.269	.092	.650	.642	.686	.576	.477		.764
10	.132	.340	.139	.586	.587	.603	.604	.417	.662	

rank. The analysis was carried through twice. At the close of the second analysis the communalities were quite similar to the communalities used in computing the first factor loadings. The communality, h^2, Table III, is the sum of the squares of the factor loadings and represents the proportion of the test variance accounted for by the factor analysis.

After two factors had been extracted, the distribution of residual correlations had a PE of .056. As the PE of a zero r when N = 113 would be .064, it would not be worthwhile to extract a third factor, especially since only ten tests were used.

The factor loadings are the amounts of the various processes or factors accounted for by the factor analysis that go to make up the total tests. A factor loading represents the correlation between a test and a factor. The residuals of the first factor are used in computing the second factor loadings as shown in Table I, Appendix B. The analysis is continued until all the factors are accounted for or until the PE of the residual correlations indicates that the remaining correlations could be due to chance. In the present study only two factors were extracted. Computation of the second factor residuals is shown in Table II, Appendix B.

TABLE III

FIRST FACTOR PATTERN SHOWING THE FACTOR LOADINGS AND COMMUNALITIES AS COMPUTED BY THE CENTROID METHOD

Test		I	II	h^2
1.	Flow	.420	-.589	.523
2.	Multiple Completion	.386	-.334	.261
3.	Paired Opposites	.580	-.361	.466
4.	Word Cross-Outs	.738	.111	.557
5.	Same-and-Opposites	.878	.339	.886
6.	Relations	.900	.201	.850
7.	Vocabulary	.798	.277	.713
8.	Hard Completion	.670	.274	.524
9.	Intelligence	.804	.444	.843
10.	Reading	.796	.203	.675

The first factor pattern showing the factor loadings and communalities as computed by the centroid method are shown in Table III. The factor loadings in Table III indicate the correlation between each test and each of the two factors. The communality, h^2, is the sum of the squares of the factor loadings and represents the proportion of the test variance accounted for by the two factors.

In order to obtain a visual picture of the results of factor analysis it is necessary to plot the test vectors in reference to the centroid axes, as in Figure 1. The number of axes is determined by the number of factors extracted.

These factors as they are represented in Table III are statistical factors and have no practical psychological meaning. In order to give psychological meaning to the factors extracted it was necessary to rotate the orthogonal axes (I and II) of Figure 1. The principle of rotation may be more easily understood by using a ball for an example. If there are spots on the cover of the ball, they do not change in their relationship to each other or to the center of the ball no matter how many times the ball is revolved or rotated. The same is true of the tests plotted in Figure 1.

The tests do not change in their relationship to each other even though the orthogonal reference axes are rotated. They may change their location in reference to any system of coordinate axes. The object of rotation is to find a system of reference axes which will locate the test vectors adequately. Or in non-mathematical language--we wish to find a few factors which will describe the tests adequately.

In Figure 1 the coordinate axes I and II represent the two mathematical factors disclosed by the factor analysis. The tests are plotted on these axes in accordance with the factor loadings. It is apparent that Tests 1, 2, and 3 fall below the horizontal axis, O-I. This appears to mean that the loadings of these tests with Factor II are negative and would be a hindrance to an individual taking these tests. A description in negative terms is not helpful so it is customary in factor analysis to rotate the axes to secure a more meaningful solution. The axes can be rotated in either direction without affecting the relative positions of the numerous tests or the communalities. Such rotation is a common procedure in analytic geometry.

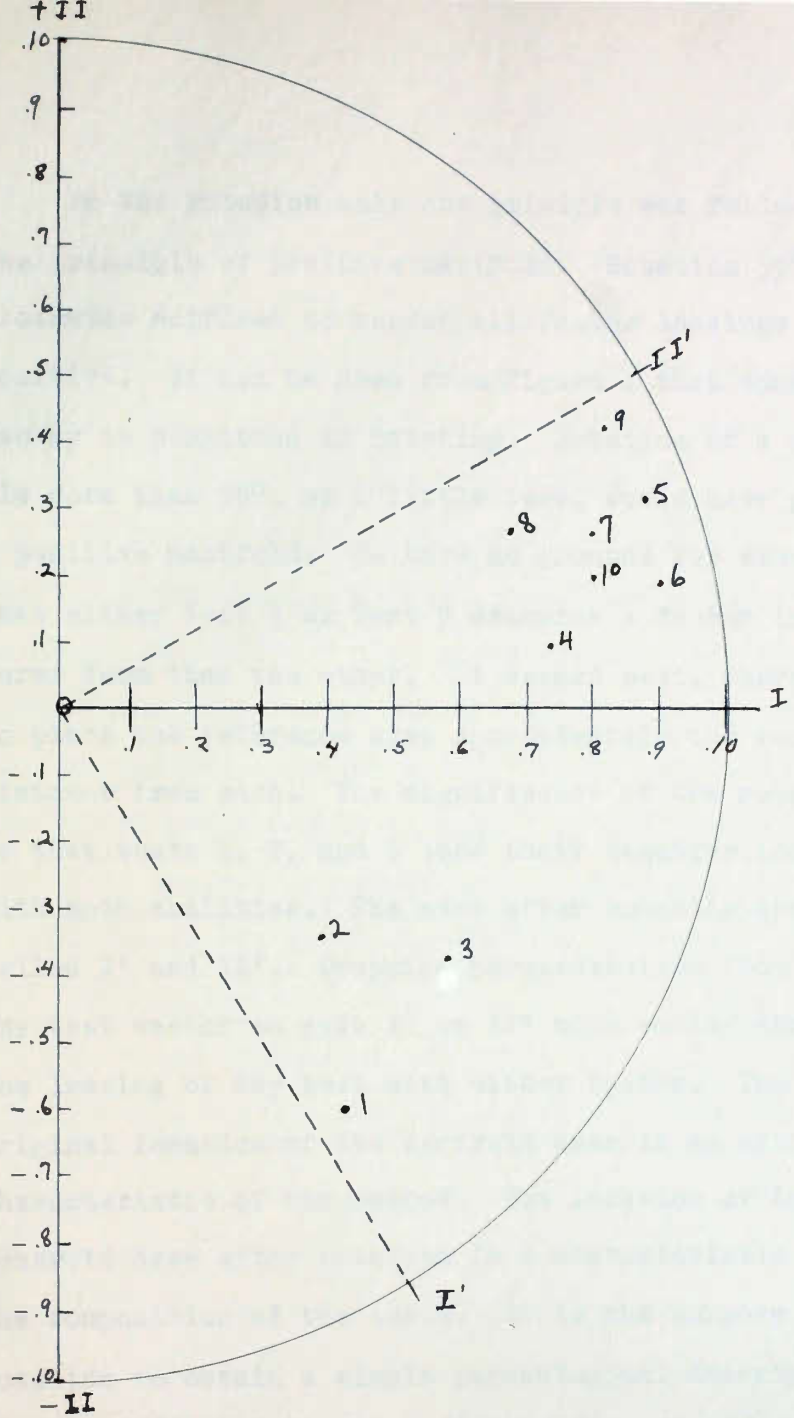

Figure 1. Showing plot of ten test vectors in reference to the centroid axes I and II. After rotation the axes are called I' and II'.

1. Flow
2. Multiple Completion
3. Paired Opposites
4. Word Cross-outs
5. Same and Opposite
6. Verbal Relations
7. Vocabulary
8. Restricted Completion
9. Intelligence
10. Reading

In the rotation only one priciple was followed: the principle of positive manifold. Rotation 59° clockwise suffices to render all factor loadings positive. It can be seen from Figure 1 that some leeway is permitted in rotating. Rotation of a little more than 59°, or a little less, would have given a positive manifold. We have no grounds for assuming that either Test 1 or Test 9 measures a factor in purer form than the other. It seemed best, therefore, to place the reference axes approximately the same distance from each. The significance of the rotation is that tests 1, 2, and 3 lose their negative loadings with both abilities. The axes after rotation are called I' and II'. Dropping perpendiculars from any test vector to axis I' or II' will easily show the loading of any test with either factor. The original location of the centroid axes is an arbitrary characteristic of the method. The location of the centroid axes after rotation is a characteristic of the composition of the tests. It is the purpose of rotation to obtain a simple psychological description of the tests. After rotation of the axes the factor loadings must be calculated in terms of the new axes. This calculation is shown in Table III,

Appendix B, and the results of this calculation are shown in Table IV.

In interpreting the rotated factor pattern one usually looks for similarities of material among the tests with high loadings in a factor. The meaning of a factor is obtained from the content of the tests which measure this factor most clearly. In order to arrive at a definite solution or interpretation of the factor pattern certain conditions must be imposed. Anastasi (1, pp. 309-310) has the following to say about such limitations:

> Any set of intercorrelations can be analyzed into factors in an infinite number of ways. In order to arrive at a determinate solution, certain "limiting conditions" must be imposed. The various current methods of factor analysis differ in their choice of limiting conditions, or postulates. It might be noted, however, that in actual practice the general results obtained by these different methods do not differ very significantly.

In the present problem the material is presumably of one kind and we therefore look for similarities in ways of handling the material, i.e. processes or functions. Test 9, Intelligence, and Test 10, Reading, are relatively complex and are not immediately helpful in the interpretation.

TABLE IV

FINAL FACTOR PATTERN AFTER ROTATION

	Test	I' (flow)	II' (selection)	h^2
1.	Flow	.721	.057	.523
2.	Multiple Completion	.485	.159	.260
3.	Paired Opposites	.607	.311	.464
4.	Word Cross-outs	.285	.689	.556
5.	Same and Opposite	.161	.927	.885
6.	Verbal Relations	.292	.874	.849
7.	Vocabulary	.174	.827	.714
8.	Restricted Completion	.110	.715	.523
9.	Intelligence	.033	.918	.844
10.	Reading	.236	.787	.675

If we look at the loadings in I' (Table IV) in the light of the hypothesis outlined in the introduction, it becomes easy to identify Factor I' as our hypothetical Factor F. The tests with the largest loadings in this factor are those which are most dependent on flow of responses. Going from Test 1 to Test 9 we see that, as the free flow of responses is restricted, the factor loadings approach zero. For greater clarification compare the contents of Tests 1, 2, and 3 with the contents of Tests 6, 7, and 8. (See Appendix A.)

By similar reasoning Factor II' can be identified as the hypothetical Factor S. The factor loadings do increase as one goes from the tests involving free flow of words to those involving the selection of the words which fit the requirements of the problem. The interpretation of the factor pattern in accordance with our hypothesis seems quite clear-cut.

A secondary purpose of the investigation was to determine to what extent Factors F and S are involved in general intelligence and in reading. Table IV shows that the Henmon-Nelson intelligence test correlates .033 with Factor F, I', flow of responses, and .918 with Factor S, II', selection of responses.

The communality of this test, h^2, is .844, an indication that the factors responsible for general intelligence have been largely accounted for. One would expect S, selection of responses, to be a more important factor in intelligence than mere flow of responses without restriction. As this test correlates fairly high with other group intelligence tests, it may be said that intelligence depends largely upon the subject's ability to select and direct responses to the more likely places, objects or ideas--that is, Factor S.

Factors F and S seem to be involved to quite an extent in the reading scores. The greatest weight is in Factor S. It is well-known that scores on tests of intelligence, vocabulary and reading are rather closely related. The correlations in Table II demonstrate these relationships again. Table IV, however, gives new information about similarities and differences in the processes operating in these three test situations.

Discussion

This is not the place for a critical discussion of factor analysis. Fortunately, several of the controversial issues in factor analysis are evaded by the simplicity of the present experimental design. For example, the rotation which is necessary to clarify the factor pattern is obvious from inspection of Figure 1. There would probably be no difference of opinion on this point. As to the independence of the factors there would be little dispute since Tests 1 and 8, which best define the two factors, show a corrected correlation of only .030. (See Table II.)

If all variance were accounted for by the two factors, the communality (h^2, Table VI) would be unity or 1. As it is some of the communalities are not very large. Extracting two factors leaves much of the variance unaccounted for. Speed of writing might account for some of the variance in the present analysis. A more complete factor analysis, determining the factorial composition of verbal tests in more detail, would use a larger number of tests--and might disclose other important processes.

Tests of verbal association and of attention span for words and concepts should be included. The purpose in the present investigation was not to do a complete factor analysis, but to determine the importance of two processes which had been postulated by experimental rather than factorial methods. In spite of the inexactnesses which have been pointed out above, the analysis indicates that the two processes considered here, calling up words, and selecting those which meet the requirements of a problem, are involved in solving problems of a verbal nature. The effects of individual differences in each of these processes on the scores of each test is indicated by the factor weights, but the writer would not claim much accuracy for the factor weights because of their dependence upon the selection of tests, selection of subjects, manner of scoring, et cetera. The relative position of the tests in terms of their loadings with each factor is probably fairly accurate.

A few points on method should be mentioned. Thurstone (16) advocates the use of simple tests the contents of which are known. In order to conform to this criterion, simple tests had to be

constructed. This required considerable labor, but the labor was well worth while as the factors can be more easily labelled and understood. Another difference between this analysis and the usual factor analysis is that a hypothesis was formulated, then an experiment planned to test the hypothesis. In many factor analyses, because of the lack of an adequate hypothesis, the results are not clear-cut.

An interesting task for the future would be to determine whether the same or similar processes are involved in solving other kinds of problems, e.g. numerical. One might expect some similarity in fitting responses to a verbal problem and to a numerical problem, but probably not in flow of verbal and numerical responses.

Is verbal ability unitary? Tests 5, 6, 7 and 8 are of the kind commonly used in factor analysis. Figure 1 shows that verbal ability is practically unitary as far as these tests are concerned. The question is whether Tests 1, 2, 3 and 4 can be ignored in a general description of mental ability. Test 4 is a test of verbal skill, somewhat similar to reading. It is definitely verbal and cannot be

ignored. The other three tests are novel tests, constructed to test a particular hypothesis. But they are certainly verbal tests. It is the writer's opinion that, if verbal ability is considered as the capacity to use verbal material in the solution of verbal problems, at least two dimensions are necessary for its factorial description.

Are other interpretations possible? Since the factor pattern was interpreted and the factors named in the light of a particular hypothesis, it is desirable to look for alternative interpretations. Reconsideration of Table IV suggests that Factor I' could be called a speed factor and Factor II' a precision factor. But these are probably merely alternative names for the factors previously called by more descriptive names. I' cannot be an unqualified speed factor because all tests were timed. It must be speed of a sort which is especially needed on Tests 1, 2, and 3.

Professor Thurstone (16) has recently discussed two verbal factors discovered by centroid analysis. V is the well-known verbal factor and W, a word-fluency factor, "is revealed by readiness in producing words to fit a specified set" (page 200).

The similarity of Thurstone's description of W and the description of F, as given in this study, is striking. It is possible that S is the usual V or verbal factor while F is the newer W or word-fluency factor. If they are the same, their independent discovery is an argument for the validity of the method.

Thurstone adds that "the recognition form of an opposites test, for example, should require little or no W, whereas a free-writing form of the same test should have high saturation in W" (p.201). This statement holds true for F as revealed in this study. Test 9, which has the least F is formally a recognition test. Psychologically, however, the flow of responses is highly restricted as only five responses are given and the task is to select the correct response. Here is an opportunity for confusion between formal and functional characteristics of a task. V and W are significantly correlated (p. 201), while F and S are not. If a wider variety of tests had been included in the present study, F and S might have been correlated.

A methodological note should be added. This investigation was planned as a test of a particular hypothesis. Then the factors were named in the light

of this hypothesis. Such a procedure introduces the possibility of bias in selecting the tests and in interpreting the factor pattern, a possibility which is greater with factor analysis than with more objective statistical tools. Nevertheless, the benefits of a hypothesis in planning the investigation and selecting the tests far outweigh the dangers of misinterpretation. Thurstone (16) points out the advantages of developing a hypothesis from a factor analysis, then testing this hypothesis by a subsequent factor analysis. The writer would add that use of factorial methods to test a hypothesis developed by different methods--in the present case, by experimental methods--has its advantages also.

Summary

The great interest shown in intelligence and problem-solving among animal and human subjects indicates that the analysis of intellectual ability is still an important problem. The experimental literature on thinking suggests that there may be two fundamental processes involved in solving all

problems: (F) the flow of various acts or responses, and (S) the selection of these responses according to the requirements of the problem. This leads to the hypothesis that individual differences in these processes would be important in determining scores on tests which include the solution of problems.

The investigation was limited to the solution of verbal problems and a battery of ten different verbal tests was given to one-hundred and twenty college students. The data were treated statistically by factor analysis (centroid method) and two independent factors were disclosed. Upon rotation (Figure 1) the two factors were identified as the hypothetical factors F and S. The importance of these factors in such common tests as vocabulary, intelligence and reading is shown by the factor weights as given in Table IV. It seems quite clear-cut that the two processes or functions suggested by the experimental literature are important in solving verbal problems and that individual differences in F, flow of responses, and S, selection of responses, are responsible for a large share of the variance in scores on verbal tests. Relations between the findings of this investigation and problem solving in general are discussed.

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3. Guilford, J. P. Human abilities. (In Psychological Review, vol. 47, 1940, pp. 367-394).

A general review of recent work on factor analysis is given. The methods and findings are discussed and criticized.

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APPENDIX A

Test # 1. Flow

Instructions: Write as many words as possible in two minutes. Do not use proper nouns.

Test # 2. Multiple Completion. Instructions: Each sentence or phrase has five blanks. Fill each blank with a word that (if used alone) completes the thought of the sentence or phrase. Example. An orange looks good round big juicy green

1. An apple is_____ _____ _____ _____ _____
2. A cat is_____ _____ _____ _____ _____
3. The horse pulled back_____ _____ _____ _____ _____
4. His appetite was_____ _____ _____ _____ _____
5. The dog was_____ _____ _____ _____ _____
6. Frogs are_____ _____ _____ _____ _____
7. The girl is_____ _____ _____ _____ _____
8. The student isn't_____ _____ _____ _____ _____
9. In a great_____ _____ _____ _____ _____
10. The point is_____ _____ _____ _____ _____
11. He has a portable_____ _____ _____ _____ _____
12. They received_____ _____ _____ _____ _____
13. I will_____ _____ _____ _____ _____a house.
14. In a_____ _____ _____ _____ _____place.
15. I can not_____ _____ _____ _____ _____her.
16. Stand_____ _____ _____ _____ _____to him.
17. Take a_____ _____ _____ _____ _____prune.
18. _____ _____ _____ _____ _____the winner.
19. _____ _____ _____ _____ _____the runaway horse.
20. _____ _____ _____ _____ _____this well.
21. _____ _____ _____ _____ _____the railing.
22. It is unkind to_____ _____ _____ _____ _____
23. This street is_____ _____ _____ _____ _____
24. He runs_____ _____ _____ _____ _____
25. A heavy_____ _____ _____ _____ _____
26. Act with_____ _____ _____ _____ _____
27. Guard the gold_____ _____ _____ _____ _____
28. You must_____ _____ _____ _____ _____
29. The folio was_____ _____ _____ _____ _____
30. Make a wide_____ _____ _____ _____ _____
31. Proceed_____ _____ _____ _____ _____
32. So they were_____ _____ _____ _____ _____ _____
33. The joints are_____ _____ _____ _____ _____
34. The passageway remained_____ _____ _____ _____ _____
35. Do not be so_____ _____ _____ _____ _____
36. Enthusiasm_____ _____ _____ _____ _____
37. Over the_____ _____ _____ _____ _____
38. Increase the_____ _____ _____ _____ _____
39. The debt was_____ _____ _____ _____ _____
40. The body was_____ _____ _____ _____ _____
41. _____ _____ _____ _____ _____the riot.
42. _____ _____ _____ _____ _____student.
43. A royal_____ _____ _____ _____ _____
44. The report was_____ _____ _____ _____ _____

45. The equator is_____ _____ _____ _____ _____
46. Perfume is_____ _____ _____ _____ _____
47. They are not so_____ _____ _____ _____ _____
48. Street cars are not_____ _____ _____ _____ _____
49. The ceremony will be_____ _____ _____ _____ _____
50. The forecast was_____ _____ _____ _____ _____
51. The exhaust tube was_____ _____ _____ _____ _____
52. Mr. Jones was injured_____ _____ _____ _____ _____

Test # 3. Paired Opposites.

Instructions: Name as many words and their opposites as possible in four minutes.

Test # 4. Word Cross-outs.

Instructions: The theme of the material below was copied from a popular magazine. Several words have been inserted into this material which definitely are incorrect in the capacity in which they are used. As you read this material, cross out the words you can find that have been inserted. For example, servile, in the first line.

Midget auto racing has developed servile a handicapping system based on the tremulous principle that the weak, not the strong, need eradicate assistance. Therefore, in doodlebug derbies, the slowest car gets the advantage of pole taxidermist position in the heats. Only in the final pallid event does the fastest car win the pole. Driving a doodlebug is a cash-and-carry proposition. The prize money, ranging from $1,500 to $2,000 paddock per night, is in the hands of the winner a few minutes oscillate after the race is over and, quicker than you can change a flat, he's off for the next stop with his still-warm midget bus strapped to the trailer behind his car. Win or lose, every driver competing gets $10 "appearance money" to cover lachrymal his expenses.

Doodlebug bugs must be insensitive to hernia odors and noises. The track sudation and its vicinity for several blocks furore in all directions smells like a childhood memory. Castor oil is an ingredient of the special insobriety lubricant used in the watch-fob motors and, long after the hieroglyphic forces of combustion have done their work, the bouquet of the delightful childhood foyer panacea still lingers on the breeze. Once the races get under way, the track becomes a cross between a gregarious boiler and a structural steel job with machine-gun obbligatos chucked in. Yet, struggling against this Niagara demarcation of noise like a cricket at a Polish wedding, the track announcer gives cadaver a rapid-fire description of the progress of the race, feeling, perhaps, that although the customers are looking directly at it and can't hear a word piscatorial of what he's saying, they won't know what's going on unless he makes his spiel.

The crazily careening little candelabrum cars are whirling canine around the track, taking the turns at almost top speed. Drivers pick their way

through holes that would scrape the slime off an eel, with the accelerator pressed down to the floor board caprice. Wheels lock, cars pivot and reverse while the rest of the field shoots dangerously past them and, if I may be vouchsafed a daring departure in phraseology, pandemonium reigns.

Despite the ear-splitting racket and the dysentery presence of the ambulance, it's a nice neighborly party and good, clean habitable fun, even though occasionally the party is turned into a wake, as it was on opening night at Castle Hill last season when Eddie Staneck, popular driver, was killed gustatory.

Although they are primarily thrill seekers, doodlebug astray addicts constitute the most patient invoice and friendly intrinsic sporting crowd that could be assembled. They aren't sharply madrigal critical like swathe baseball fans; impatient sycophant with delay in dishing out the inaugerate mayhem, like boxing bugs; sophisticated and digit snooty like a polite nauseous tennis gallery or maudlin like a football crowd when the flasks are briny low. They sit patiently for a half-hour at a time between races with nothing to entertain them but a spot of pastoral thumb-twiddling, the delightful castor-oil bouquet on the evening accolade breeze and the trite witticisms of the bovine announcer, all of which they have card indexed. When the glib gent at the mike asks them to give "good old Gus, here" a big aerodrome hand, they obey him like the citizen of a totalitarian state who has received his escutcheon voting instructions from his dictator. Let the announcer remind them refreshments may be malady bought and they call on the vendors in a body for franks and pop.

Finally, along about 11 o'clock, when the winner flashes across the slough line like a streak of hybrid rainbow lightning, the crowd of average-priced reciprocal car owners, their mates and their young, snap out of the trance in which they had demonstrated to themselves the everlasting truth of the doctrine of the transmigration of living souls. No longer the dare-devil emanate drivers they had been, in fancy, during their few hours embryo of escape from the humdrum reality of their dull lives, they become average citizens again, pile fallacious into the old family rattle trap and proceed homeward, carefully obeying all paresis speed laws en route. For although a doodlebug will never get them, they know a motorcycle elocution cop will, if they don't watch out embellish.

Test # 6. Verbal Relations.

Directions: This is a test of your ability to see corresponding relationships. For example, (a) tar--
--throw. The problem is to place a word in the blank that will mean tar and at the same time mean throw--
"pitch" is the word. Another example is (b) saloon--
--rod, "bar" is the word in this case. Another problem is to pick the number of the proper word and place it in the blank. For example, sun is to day as moon is to_____1. sea, 2. dog, 3. night, 4. winter. Of course the number 3 would be written in the blank.

1. Irksome is to wearisome as engrossed is to_____
   1. enhanced, 2. occupied, 3. apathy, 4. dilapidated.
2. fee-- --end
3. Artisan is to amateur as concur is to_____
   1. dissent, 2. appeal, 3. confer, 4. agree.
4. plank-- --meals
5. Ostensible is to pretended as insidious is to_____
   1. frank, 2. candid, 3. treacherous, 4. notable.
6. jar-- --able
7. Lifeless is to inanimate as qualified is to_____
   1. disinterested, 2. new, 3. competent, 4. animosity.
8. poster-- --beak
9. Superior is to excel as reclining is to_____
   1. devoted, 2. calculate, 3. conserve, 4. recumbent.
10. case-- --fight
11. Sect is to cult as fanatic is to_____1. fanciless, 2. enthusiast, 3. liberal, 4. pinnacle.
12. vessel-- --face
13. Indict is to parole as ensnare is to_____1. deceit, 2. involve, 3. reward, 4. conserve.
14. arm-- --branch
15. Docile is to submissive as contradict is to_____
    1. gainsay, 2. persuade, 3. reward, 4. chastise.
16. bird-- --frolic
17. _____is to prophecy as advocate is to endorse.
    1. postulate, 2. truncate, 3. apall, 4. prediction.
18. clean-- --search
19. Madrigal is to song as dearth is to_____1. whim,
    2. molest, 3. audible, 4. scarcity.
20. seed-- --hole
21. Holstein is to cow as Percheron is to_____.
22. pip-- --look

23. Feline is to cat as canine is to_____.
24. jail-- --write
25. Logic is to reasoning as loll is to_____1. ludicrous, 2. perspicuous, 3. lounge, 4. lucifer.
26. scrape-- --foot
27. Rustic is to unadorned as destruction is to_____ 1. sabotage, 2. sacrificial, 3. expulsion, 4. credulous.
28. conceal-- --skin
29. Crevice is to fissure as chevoit is to_____1. segment, 2. inane, 3. delirious, 4. cloth.
30. fasten-- --tress

APPENDIX B

TABLE I

INTERCORRELATIONS OF THE 10 TESTS AS OBTAINED FROM THE CORRECTED CORRELATIONS GIVEN IN TABLE II, AND THE COMPUTATION OF THE FIRST FACTOR LOADINGS

1	**.529**	.415	.573	.310	.297	.275	.181	.030	.135	.185
2	.415	**.289**	.299	.211	.249	.193	.203	.123	.308	.403
3	.573	.299	**.430**	.387	.517	.567	.411	.469	.153	.238
4	.310	.211	.387	**.583**	.525	.650	.577	.452	.750	.695
5	.297	.249	.517	.525	**.816**	.811	.829	.659	.730	.686
6	.275	.193	.567	.650	.811	**.868**	.688	.754	.821	.642
7	.181	.203	.411	.577	.829	.688	.739	.548	.665	.717
8	.030	.123	.469	.452	.659	.754	.548	**.570**	.562	.505
9	.135	.308	.153	.750	.730	.821	.665	.562	**.712**	.764
10	.185	.403	.238	.695	.686	.642	.717	.505	.764	**.714**
Σr_{ka}	2.930	2.693	4.044	5.140	6.119	6.269	5.558	4.672	5.600	5.549=48.57
k_1	.420	.386	.580	.738	.878	.900	.798	.670	.804	.796

Formula for computing the First Factor Loadings: $a_1 = \frac{\Sigma r_{ka}}{\sqrt{\Sigma r}}$

a_1 = the factor loading in test a.

Σr_{ka} = the sum of the coefficients in the column for test a.*

Σr = the sum of all the values of r in the table.

*The subscript ka is interpreted as follows: The first letter k is a general notation and stands for the row in which the r appears. The second letter indicates the column in which r appears. Σr_{ka} then means the sum of the r's in the column for test a, or in this case, column 1.

TABLE II

FIRST FACTOR RESIDUALS AND COMPUTATION OF SECOND FACTOR LOADINGS

		+	-1	+	-2	+	-3	+	+4	+	+5	+	+6	+	+7	+	+8	+	+9	+	+10
+	- 1	+ -	+.329	+ -	+.253	+ -	+.329	+	-.000	-	+.072	-	+.103	-	+.154	-	+.251	-	+.203	-	+.149
+	- 2	+ -	+.253	+ -	+.253	+ -	+.075	-	+.074	-	+.090	-	+.154	-	+.105	-	+.135	-	+.002	+	-.096
+	- 3	+ -	+.329	+ -	+.075	+ -	+.329	-	+.041	+	-.008	+	-.045	-	+.052	+	-.080	-	+.313	-	+.124
+	+ 4	+	-.000	-	+.074	-	+.041	+	+.157	-	-.123	-	-.014	-	-.012	-	-.042	+	+.157	+	+.108
+	+ 5	-	+.072	-	+.090	+	-.008	-	-.123	+	+.240	+	+.132	+	+.240	+	+.182	+	+.136	+	+.099
+	+ 6	-	+.103	-	+.154	+	-.045	-	-.014	+	+.132	+	+.154	-	-.030	+	+.151	+	+.097	-	-.074
+	+ 7	-	+.154	-	+.105	-	+.052	-	-.012	+	+.240	-	-.030	+	+.240	+	+.013	+	+.023	+	+.082
+	+ 8	-	+.251	-	+.136	+	-.080	-	-.042	+	+.182	+	+.151	+	+.013	+	+.251	+	+.023	-	-.028
+	+ 9	-	+.203	-	+.002	-	+.313	+	+.157	+	+.136	+	+.097	+	+.023	+	+.023	+	+.313	+	+.124
+	+10	-	+.149	+	-.096	-	+.124	+	+.108	+	+.099	-	-.074	+	+.082	-	-.028	+	+.124	+	+.149
Σr_{ka}			+1.843		+1.046		+1.130		+ .346		+1.060		+ .628		+ .867		+ .857		+1.391		+ .637
k_2			- .589		- .334		- .361		+ .111		+ .339		+ .201		+ .277		+ .274		+ .444		+ .203

TABLE III

CALCULATION OF NEW FACTOR LOADINGS AFTER ROTATION OF 59°

Test	I	x	COS	=	a	II	x	SIN	=	b	a	-	b	=	I'
1.	.420		.515		.216	-.589		.857		-.505	.216		-.505		.721
2.	.386		.515		.199	-.334		.857		-.286	.199		-.286		.485
3.	.580		.515		.299	-.361		.857		-.309	.299		-.309		.607
4.	.738		.515		.380	.111		.857		.095	.380		.095		.285
5.	.878		.515		.452	.339		.857		.291	.452		.291		.161
6.	.900		.515		.464	.201		.857		.172	.464		.172		.292
7.	.798		.515		.411	.277		.857		.237	.411		.237		.174
8.	.670		.515		.345	.274		.857		.235	.345		.235		.110
9.	.804		.515		.414	.444		.857		.381	.414		.381		.033
10.	.796		.515		.410	.203		.857		.174	.410		.174		.236

Test	II	x	COS	=	c	I	x	SIN	=	d	c	+	d	=	II'
1.	-.589		.515		-.303	.420		.857		.360	-.303		.360		.057
2.	-.334		.515		-.172	.386		.857		.331	-.172		.331		.159
3.	-.361		.515		-.186	.580		.857		.497	-.186		.497		.311
4.	.111		.515		.057	.738		.857		.632	.057		.632		.689
5.	.339		.515		.175	.878		.857		.752	.175		.752		.927
6.	.201		.515		.103	.900		.857		.771	.103		.771		.874
7.	.277		.515		.143	.798		.857		.684	.143		.684		.827
8.	.274		.515		.141	.670		.857		.574	.141		.574		.715
9.	.444		.515		.229	.804		.857		.689	.229		.689		.918
10.	.203		.515		.105	.796		.857		.672	.105		.682		.787

Formula-- I' = I x COS - II x SIN
II' = II x COS + I x SIN

I = factor loading
II = other factor loading
COS = cosine of angle of rotation
SIN = sine of angle of rotation
a = product of I times the COS
b = product of II times the SIN
c = product of II times the COS
d = product of I times the SIN
I' and II' are the new factor loadings